On the Dynamic and Robust Management of Multi-Product Perishables, Multi-Echelon Demand Networks

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***Abstract*— This paper addresses a preliminary study of perishable inventory management (PIM). The aim is the development of a robust and efficient control algorithm tacking into account the stock deterioration and its replenishment subject to a delay from a remote supply source and reacting significantly faster to the consumer's demand, which is considered as unknown disturbance. Here, instead of model-based control strategies, through a continuous, local identification of the level variations a model-free control that does not need any prior knowledge of the supply chain behaviors is proposed. The developed control policy permits to take into account both the transportation losses which could appear between the suppliers and the final customers and accounts for decay of goods stored in warehouse (perishing inventories) preventing from exceeding the facilities capacities.**

***Index Terms*–Perishable inventory management, algebraic estimation, model-free control, bullwhip effect, delay systems**

I. Introduction

Perishable Inventory Control (PIC) represents a salient problem in the frame of dynamic supply chain management. In the simple inventory control, the objective is the design of a system to monitor different levels of the studied supply chains continuously and periodically assuming unlimited lifespans for the different products [11]. Nevertheless, most of the products are characterized by different shelf-lives and become unfit for consumption. Therefore, PIC problems are concerned by an efficient design of inventory plan tacking into account (See e.g., [15] and the references therein for a more precise discussions), the proper time period to place orders, the size of the orders, and the allocation of products with different shelf-lives to demand. As confirmed in [12], the worth of the wasted food is in the billions of dollars annually. Perishable products concern not only food but also blood, pharmaceuticals products, chemicals, cut flowers which are with a limited shelf-time.

Effective management and control of such systems become of utmost importance. In this frame several approaches are devoted to the PIM problem were the unknown disturbance, i.e., consumers' demand are taking into account. Simulation approaches, Markov dynamic programming, methods stemming from optimization techniques are among the several approaches used in PIM problems such as the study of blood bank [13], coordination of inventory management and material handling in hospital structures [3] (See e.g., [15] and the references therein, for a review of most simulation and optimization techniques used in the PIM problems).

Others approaches stemming from automatic control theory have largely been applied. Nevertheless, the main difficulty in developing control schemes for perishable inventories stems from the necessity of conducting exact analysis of product lifetimes. The design problem becomes cumbersome in the situation when the product demand is subject to significant uncertainty and inventories are replenished with no negligible delay, which frequently happens in modern supply chains.

Control problems for perishable products have been extensively studied in the literature [10], [15], [4], [20], [5], [17], to answer the typical question: *What are the optimal time and volume of the order, that best satisfy the customer demand, while accounting for product perishability?* [11]. When manufactured products are perishable, accounting for demand variations, the delay (due to transportation and manufacturing) is critical for correctly determine the safety stock level. The main difficulty in developing control schemes for perishable inventories stems from the necessity of conducting exact analysis of product lifetimes. The design problem becomes cumbersome in the situation when the product demand is subject to significant uncertainty and inventories are replenished with no negligible delay, which frequently happens in modern supply chains. In such conditions, in order to maintain high service level and at the same time keep stringent cost discipline, when placing an order, It is needful to take into account both the demand during the procurement latency and the stock deterioration in that time. The controller gain should be adjusted to obtain fast reaction to demand transitions. From an operational perspective, manufacturing systems are confronting a lack of strategies that consider the nature of perishable products, while maintaining economic efficiency. Indeed, perishable products are characterized by a limited shelf life, which can be either deterministic or random [11]. The literature of operations management for perishable products reports a correlation between the shelf-life of the products and machine degradation. [1] develop robust and computationally efficient supply chain management strategy ensuring fast reaction to the demand variations for periodic review perishable inventory systems. For that purpose, they apply a sliding mode approach and they propose a new discrete time warehouse management strategy. [16] consider the problem of designing an efficient supply strategy for logistic systems with perishable goods. The stock deteriorates exponentially, and is replenished with delay from a remote supply source. As opposed to the previous approaches based mainly on heuristics and static optimization, they apply formal design methodology of sliding-mode control and discrete-time dynamical optimization. [4] have considered Model Predictive Control (MPC) in an optimization role. In this frame, MPC is used to manage a multi-product, multi-echelon production and distribution network by a centralized structure which can provide better performances than a decentralized structure in the case studies examined in [20]. [5] propose a mathematical model for manufacturing systems that incorporates uncertain processing time, delayed produced items and has deteriorating items with a given constant rates.

Faced with the difficulties of modeling all the phenomena of the complex and meshed supply chain systems, non-model-based and particularly, the model-free control (MFC) law seems to be a solid alternative to deal with perishable inventory control.

This paper which differs quit a lot from the model-based control strategies is organized as follows. Section II, describes briefly the main principles of model-free control. The mathematical model used for simulation purposes is described in Section III. Section IV, is devoted to the application of model-free control where numerous convincing computer simulations are presented and discussed for a study case of a two-product perishables, three-echelons, six-nodes demand networks manufacturing supply chain. Finally Section V, gives some concluding remarks and perspectives for future research.

II. Model-free control: Main principles

Model-free control and its corresponding intelligent controllers have been developed by M. Fliess and C. Join (See e.g., [8], [7], for a deep and thorough presentation of the MFC concept). Notice. that the proposed model-free control approach has demonstrate its usefulness in many situations like the compensation of severe non-linearities, and the compensation of time-varying reference signals.

In addition, the corresponding intelligent controllers are much easier to implement and to tune. Notice that, although the MFC is relatively a new concept, several successful concrete applications in different fields have been developed and most of them have actually been implemented [8], [9], [2], [14], [18], [19].

A. *The phenomenological model*

In the current application of perishable inventory control, the unknown studied supply chain is restricted to a SISO (Single-Input, Single-Output) system because the objective is to maintain different levels at desired v¡alues thanks to the production rate at different nodes of the plant. Therefore, following the MFC concept, the unknown expression of the SISO supply chain is replaced by a phenomenological first order model

$$\dot{y}(t) = F(t) + \alpha u(t) \tag{1}$$

where, y and u represent, the output and control variables, respectively, $\dot{y}$ represents the first derivative of the output variable, F which is a time-varying quantity subsumes not only the unknown internal structure, but also the external disturbances. This quantity is estimated, $\alpha \in \mathbb{R}$ is chosen by the practitioner such that $\dot{y}$ and αu are of the same magnitude. Therefore, α does not need to be precisely estimated. The model (1) is only valid during a short time lapse that must be continuously updated and then said an "ultra-local" model.

B. *The corresponding Intelligent controllers*

Assume that F is estimated and close the loop with the following proportional (P) controller or i-P controller [9]:

$$u = -\frac{F - \dot{y}_{ref} + K_P e}{\alpha} \tag{2}$$

where $y^\star$ represents the output reference trajectory; $e = y_{ref} - y$ is the tracking error; and K_P is a usual tuning gain. The i-P controller (2) is compensating the poorly known term F. The control of the studied supply chain therefore boils down to the control of an elementary pure integrator. Combining (1) and (2) yields

$$\dot{e}(t) + K_P e(t) = 0 \tag{3}$$

where F does not appear anymore. Thus $lim_{t\to\infty} e(t) = 0$ if $K_P > 0$. This local stability property proves that the tuning of K_p is straightforward.

Remark In order to estimate the first derivative of the output variable y, recent algebraic parameter identification techniques can be used [6].

C. Online estimation of F

For the estimation of F, rewrite (1) in Laplace domain,

$$sY = \frac{\Phi}{s} + \alpha U + y_0 \quad (4)$$

where

- Φ is a constant,
- y_0 is the initial condition corresponding to the time interval $[t-\tau, t]$.

In order to get rid of y_0, multiply both sides by $\frac{d}{ds}$

$$Y + s\frac{dY}{ds} = -\frac{\Phi}{s^2} + \alpha\frac{dU}{ds} \quad (5)$$

For smoothing the noise, multiply both sides by s^{-3} which in time domain yields

$$F = -\frac{6}{\tau^3}\int_{t-\tau}^{t} \left((\tau - 2L)y(L) + \alpha L(\tau - L)u(L)\right) dL \quad (6)$$

where L is quite small and depends on the sampling period as well as the noise intensity.

III. Dynamic Modeling and Control

Although the model-free control does not require any a priori model, a simulation model is built to reflect the system dynamics governed by the proposed control policy. The objective is to assess the system behavior over the planning horizon and generate outputs that will serve to compute the total cost.

A. Production-inventory system: A mathematical model

Assume that the on-hand stock $y(t)$ used to fulfill the market demand $d(t)$ deteriorates at a constant rate σ, $0 \leq \sigma < 1$ when kept in the distribution center warehouse. Denoting the quantity ordered from the supplier at time t by $u(t)$, the stock balance equation can be written in the following way,

$$\frac{dy}{dt} = \begin{cases} -\sigma y(t) + \rho u_r(t) - d(t), & t \geqslant \theta \\ -\sigma y(t) - d(t), & 0 \leq t < \theta \end{cases} \quad (7)$$

where $u_r(t) = u(t-\theta)$ is the received shipment, θ is the factory throughput time and ρ ($0 \leq \rho \leq 1$) is the yield.

Physical limitations (bounds in the inventory level $y(t)$, flow rates $u(t)$ and the market demand $d(t)$) must be taking into account as follow:

$$\begin{cases} y_{min} \leq y(t) \leq y_{max} \\ u_{min} \leq u(t) \leq u_{max} \\ d_{min} \leq d(t) \leq d_{max} \end{cases} \quad (8)$$

Notice that the concept of model-free control does not requires a precise configurations of the manufacturing supply chain, the above model of the supply chain networks is used only in order to test the proposed controllers.

B. Multi-product perishables, multi-echelon demand networks model-free control

For implementation, it is strongly advisable to discretize (6) and filter $\hat{F}$ using a classic second filter [9]. Then the ultra-local model for each node of the multi-product perishables, multi-echelon demand networks is given by (9):

$$\dot{y}_i(t) = F_i(t) + \alpha_i u_i(t) \quad (9)$$

A fully decentralized structure (one controller for each node) is required to manage the constraints due to the interconnections of the nodes, the release rate capacities, and the shipping capacities. Then, for each node, the corresponding Intelligent controller (iP) is given by (10):

$$u_i = \frac{1}{\alpha_i}\left(-\hat{F}_i(t) - \dot{y}_i^\star(t) + K_{P_i} e(t)\right) \quad (10)$$

where i, denotes each of n nodes, $i = 1, ..., n$.

IV. Application to Real Industrial Example

The main objective of the supply chain control is to keep inventory level y_i ($i = 1, ..., n$ with $n = 6$) to its desired value $y_i^\star$ (the safety stock level) when manufactured products are perishable deteriorates at a constant rate σ_i , while the customer demand variations must be satisfied respecting physical constraints given by (8).

A. Case study

In order to assess the efficiency of the proposed approach, consider a two-product perishables (A and B), three-echelons (I: Factories, II: Warehouses and III : Retailers), six-nodes demand networks manufacturing supply chain depicted in Fig. 1.

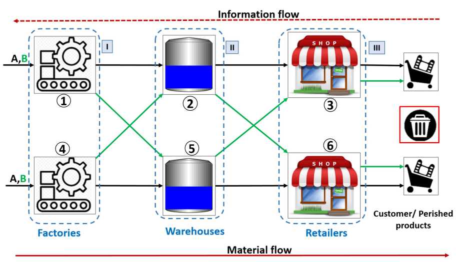

Fig. 1. two-product perishables, 3-echelons, six-nodes system

The stock balance equation in each node (7), can be written then:

$$\dot{y}_i(t) = \begin{cases} -\sigma_i y_i(t) + \rho_i u_i(t-\theta_i) - d_i(t), & t \geqslant \theta_i \\ -\sigma_i y_i(t) - d_i(t), & 0 \leq t < \theta_i \end{cases} \quad (11)$$

TABLE I

NUMERICAL VALUES

Echelons	Node	$y_i^\star(A,B)$	$y_{0,i}(A,B)$
Echelon I :	1	$(3,2).10^4$	$(4,2.5).10^3$
Factories	4	$(3.5,2.5).10^4$	$(4.5,3).10^3$
Echelon II :	2	$(1.5,1.2).10^4$	$(2,3).10^3$
Warehouses	5	$(1,0.8).10^4$	$(3,1).10^3$
Echelon III :	3	$(1.2,1).10^4$	$(1,0.5).10^3$
Retailers	6	$(1,0.8).10^4$	$(1,0.8).10^3$

with $i = 1, ..., 6$.
The numerical values of different parameters used in this paper are summarized in table I. $u_{i,min} = 0$; $\alpha_i(A,B) = 10$; $\theta_{\{1,4\}} = 4$; $\theta_{\{2,5\}} = 3$; $\theta_{\{3,6\}} = 2$, $\sigma_i = 0.9$; and $\rho_i = 1$. $u_{i,max}(A,B) = (13,9.3).10^4$, $(15,12).10^4$, $(1.7,4).10^4$, $(5,4).10^4$, $(1.9,1.6).10^4$ and $(1.7,2).10^4$ for the different echelons In addition: $y_i^\star$ in MT: Mega tonnes; $u_{i,max}$ and $u_{i,min}$ in MT/days; and θ_i in days.

Fig. 2 represents the total demand of customers $d_A(t)$ and $d_B(t)$ respectively for the products A and B. The customer's demand signal is stochastic but bounded: $d_{A,max} = 8500$, $d_{B,max} = 6500$ and $d_{A,min} = 0$, $d_{B,min} = 0$ in MT/days;

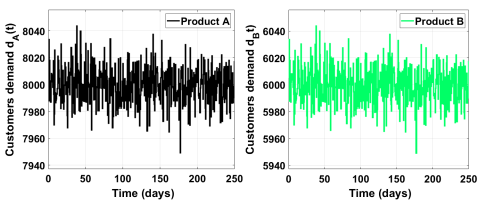

Fig. 2. Customers demand of the two-product perishables

B. Numerical results

Consider, first, the information flow, i.e, from the customers, via retailers (echelon 3) and warehouses (echelon 2) to factories (echelon 1).

1) Numerical results for retailers (echelon 3): According to the direction of information flow, retailers are the echelon directly related to customers demand $d_A(t)$ and $d_B(t)$ in the studied supply chain. The supply chain is modeled as a serial process where each node in each echelon orders goods from its/their immediate supplier(s). Then the immediate suppliers of the retailers in this case study are warehouses and for warehouses are factories. Then, the production rate $u_3(t)$ and $u_6(t)$ of echelon 3 are the warehouse demand (echelon 2) and the production rate $u_2(t)$ and $u_5(t)$ are the demand for factories (echelon 1).

Fig. 3 represents the inventory level and production rate in node 3 for perishables products A and B, respectively. The obtained results show that the inventories level ($y_{3,A}(t)$, $y_{3,B}(t)$) converge to their desired value ($y_{ref3,A}(t)$, $y_{ref3,B}(t)$) while respect of physical constraints corresponding to their maximal production rate ($u_{3,A}(t)$, $u_{3,B}(t)$). Note that the MFC (iP) stands out by its speed to reach steady state and permits to take into account the delay times θ_3 and also allows to ensure that ordered quantities of goods $u_3(t)$ are always non-negative and upper bounded. The same results are seen in Fig. 4 for the node 6.

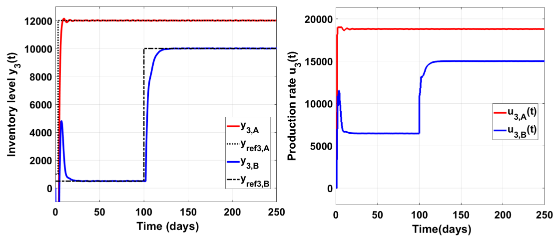

Fig. 3. Inventory level and production rate in node 3.

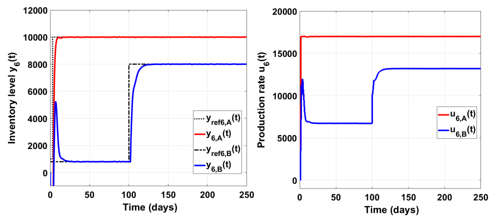

Fig. 4. Inventory level and production rate in node 6

2) Numerical results for warehouses (echelon 2): Fig. 5 and Fig. 6 represent the inventories levels and production rate in nodes 2 and 5 of echelon 2, for perishables products A and B. These figures, show some fluctuation and overshoots in inventories levels of products A and B, $y_2(t)$ and $y_5(t)$ in response to abrupt demand changes ($u_3(t)$ and $u_6(t)$), the deteriorate constant σ and the delay time θ. In $y_{2,B}(t)$ and $y_{5,B}(t)$, the stock are depleted $y(t) = 0$ at some interval of time such as $t \in [100-105]$ days for example which implies the not full demand satisfaction.

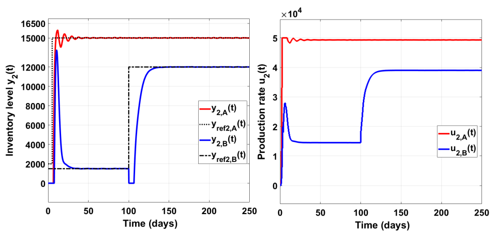

Fig. 5. Inventory level and production rate in node 2

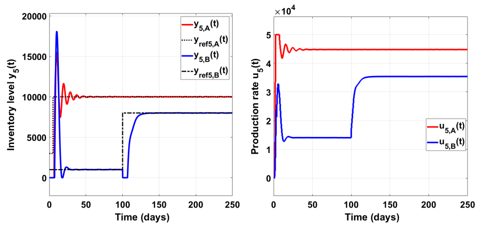

Fig. 6. Inventory level and production rate in node 5

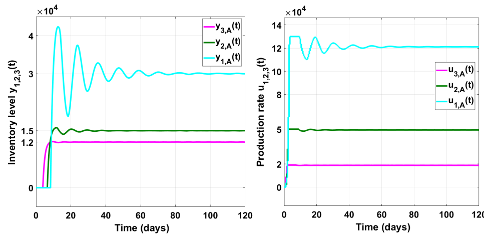

Fig. 9. Inventory level and production rate in node 1, 2, 3

3) Numerical results for factories (echelon 1) : Fig. 7 and Fig. 8 depict the inventories levels and production rates evolutions in nodes 1 and 4 of echelon 1, for perishables products A and B, respectively.

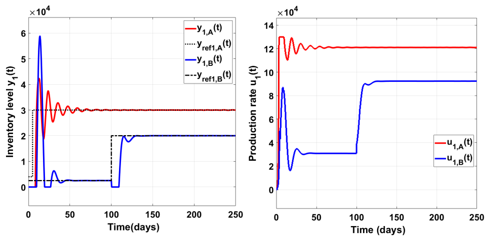

Fig. 7. Inventory level and production rate in node 1

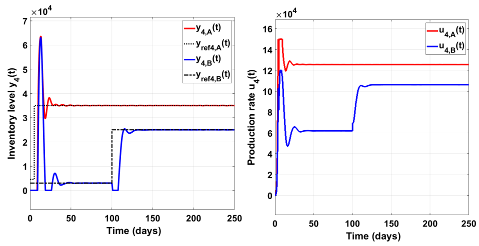

Fig. 8. Inventory level and production rate in node 4

The same results as for echelon 2 are observed in Fig. 7 and Fig. 8, but with more higher fluctuations. Indeed, the variability in the ordering patterns increases downstream in the supply chain, from the customer the factory. This is a typical example of so-called "bullwhip effet" phenomenon often encountered in supply chain systems.

The focus is made on the inventories levels evolutions for product A and the production rates in nodes 1, 2, 3 (See e.g. Fig. 9) where the fluctuations are increased and amplified.

4) Analysis of the influence of $\sigma_3(t)$ in node 3 with product A: It is necessary to highlight the importance of considering shelf life $\sigma(t)$ variability in inventory control, that ignoring it can affect incurred losses. Thereby, over 4 periods of 250 days, consider the variation of $\sigma_3(t)$ as depicted in (12):

$$\begin{cases} 0 \leq t \leq 250, \ \sigma_3 = 0.8 \\ 250 < t \leq 500, \ \sigma_3 = 1 \\ 500 < t \leq 750, \ \sigma_3 = 0.6 \\ 750 < t \leq 1000, \ \sigma_3 = 0.9 \end{cases} \quad (12)$$

with the following values; $y_{ref,3}(A) = 12000$, $y_{0,3}(A) = 1000$ and $u_{3,max}(A) = 19000$.

Fig. 10 represents the Inventory level and production rate in nodes 3 of echelon 3, for perishable product A and a variable shelf life. Notice that the fluctuation and overshoots in inventories levels $y_3(t)$ and production rate $u_3(t)$ increase when $\sigma_3(t)$ decreases. Moreover, when $\sigma_3(t) \approx 1$, the number of products ordered increases with a low inventory level, i.e., less product loss.

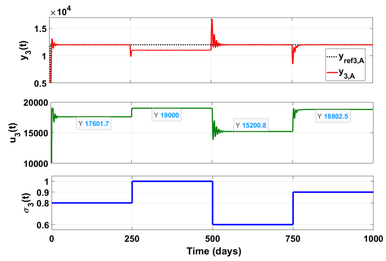

Fig. 10. Influence of the deteriorate constant $\sigma_3(t)$ in node 3

V. Conclusions

The proposed in this paper, model-free control approach allows to bypass any dynamic mathematical model for the supply-chain management. It has demonstrated its capability to manage inventory in supply/demand network on a real study case, two-product perishables, three-echelons, six-nodes demand networks manufacturing supply chain. The results are discussed and shown that the controllers are stable and appropriate for controlling inventory in supply chain while ensuring the synchronization of whole system. MFC offers the flexibility in terms of the information sharing, network topology, and constraints that can be handled.

In order to solve the stability problems related to the unavoidable delay, which is the major obstacle in the considered application in the feedback loop, further works based on the use of Smith predictor, will be developed in order to compensate this delay. Although Smith predictors assume the knowledge of the this delay and the system's models, the developed approach will treats the delay as unknown and then algebraic methods of identification will be used in order to estimate this parameter in the frame of Smith predictor. Such so-called Smith predictor with the ultra-local model (SP-ULM) seem to be more efficient to mitigate the bullwhip effect. In some echelons, the conducted study has demonstrate that the stock is depleted at some interval of time, which implies the not full demand satisfaction in that interval of time, from the readily available resources and 100% service level. However, the designed controller generates overshoots at the output $y(t)$ in response to abrupt demand changes (nodes, $\ldots$). The ideal case for supply chain is to produce the exact amount of products needed by customers. By using the forecast, the net stock can be reduced compared to the net stock obtained without a forecast. Therefore, in order to overcome the problems related to the increased storage space due to the stock level overshoots, algebraic approaches will be integrated in order to perform demand forecasting which will allows an anticipatory character of the control. Then, a natural extension of this work is to incorporate the Smith Predictor with Ultra-Local Model (SP-ULM) and the demand forecast using algebraic approaches in the control for supply system with perishable goods.

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