Inventory Control in Supply Chain: a Model-Free Approach

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Abstract:
Inventory control in the frame of supply chain systems represents a challenging problem due to their dynamical behavior, ranging from fast changing demand, transport and delivery delays and production phenomena like bullwhip effect. Nevertheless, modeling such systems in order to capture all its components and to design a robust and dynamic management, remains an open problem. In this paper, a model-free control which permits to design several control strategies without need to any exact model, is proposed. Several numerical simulations as well as the conducted comparative studies show the relevance of the proposed approach. In addition, the paper considers the case of a poor knowledge of the supply chain model and introduce an exact and fast deterministic method which is of a algebraic flavor in order to estimate the parameters of the model.

Keywords: Model free control, supply chains, inventory control, lead time, algebraic estimation.

1. INTRODUCTION

Supply chain systems represent a set of meshed and complex networks that characterize the behavior, integration and relations between the set of entities (nodes) in a network such as: producers, suppliers, warehouses, transports, distributors and customers. The whole of these nodes exchange materials and information flow. Inventory management represents a main issue in logistic systems planning and operations. Controlling such nodes permits, among others, to determine the good timing of issuing replenishment order quantity which can be subject to demand uncertainties and/or other system parameters. Therefore, improved inventory management contributes to increased revenues, lower costs and greater customer satisfaction. In the framework of supply chain systems, the coordination and the synchronisation of information and material flows becomes an important issue in order to optimize the use of resources. These issues introduce a new challenges including several environment, social and societal aspects, with the advent of sustainable development approaches, Wang et al. (2019).

Recalling that supply chain is a system which is a characterized by unavoidable dead times or lead/throughput times. Such random delays are due to phenomena related to transport and production for example. In addition, integrative dynamics at the inventory levels lead to the appearance of instability, inventory drift (e.g., the difference between the inventory target and actual inventory level, Salcedo et al. (2013)) and bullwhip effect [1] (the variability in the ordering patterns often increases as we move up into the chain, from the customer towards the suppliers and factory, for example, Garcia et al. (2012)). All these phenomena are source of supply chain inefficiencies due to its harmful consequences including storage, shortage, labour and transports costs, Ponte et al. (2017). As manufacturing enterprises move ahead it is increasingly difficult to compete on a global scale without strong replenishment policies.

Several studies for dynamic inventory management have been considered. Approaches stemming from control theory have been successfully applied to production inventory, replenishment policies and dynamic supply chain management. Dejonckheere et al. (2003) have analyzed the effect of the replenishment policies focused on the bullwhip effect estimation and elimination. The effect of inventory variability with linear control theory has been developed by Hoberg et al. (2007). They proposed a study of a two-echelon supply chain with a stationary demand pattern under the influence of three inventory policies. Rodríguez-Angeles et al. (2009) have considered the supply chain as a set of producers and non-producers nodes with different dynamics, the main objective is to maintain a desired inventory level while the demand of customers is satisfied and the incoming and outgoing flows are synchronized. Garcia et al. (2012) have used an Internal Model control (IMC) scheme with online identification of lead times based on a multi-model schema. The identified values are then used to adjust the delay compensation in a decentralized IMC. Schwartz and Rivera (2010) have

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[1] Notice that the literature does not seem to contain any clear-cut definition of this effect.

introduced two and three degrees of freedom feedback feedforward IMC as well as model predictive control as a novel inventory replenishment policy in supply chain and its applications to the control of a single echelon. Centralized IMC approaches (two-degrees of freedom) where inventory target tracking and disturbance (demand) rejection in the inventory control, treated separately, have been developed by Salcedo et al. (2013). A comparative study with the decentralized IMC approach show the efficiency of the centralized IMC, which enhances the behavior with respect to all the above objectives as well as bullwhip effect in supply chain systems. A decentralized approach based on differential flatness concept was proposed by Hamiche et al. (2017), for dynamic supply chain systems with parametric uncertainties. Several other approaches dealing with stochastic inventory control are summarized in the literature review provided by Ma et al. (2019).

Nevertheless, all the above approaches are model-based where the representation of the studied supply chain rests on some empirical assumptions that simplify their complexity. Therefore the efficiency of the control algorithm is closely related to the quality of the used mathematical model. In this paper, our presentation is quite different, since it is based on the recently introduced *model-free control* concept. Indeed, model-free controllers to supply chain systems seems to be a promising research direction considering supply chains complexity.

Our paper is organized as follows. Section 2, recalls briefly the model-free control setting and its related intelligent controllers. Section 3 presents, the dynamical model of the supply chain and some controllers, from literature, used in the comparative study. Numerical simulations borrowed from Rodríguez-Angeles et al. (2009), and some comparative studies are discussed in Section 4. Section 5 considers the case of a supply chain described by a restricted model and provides an online parametric estimation method based on numerical differentiation. Finally, section 6 gives some concluding remarks and perspectives for future research.

2. MODEL-FREE CONTROL

Model free control may be viewed as a contribution to *intelligent* PID controllers (*iPIDs*). It was introduced few years ago by Fliess and Join (2013) with several successful concrete applications in different fields such as: the control of robot manipulator, Abouaïssa and Chouraqui (2019), on motor throttle, Join et al. (2008), on lateral/longitudinal control for automotive vehicles, Menhour et al. (2011), on fault detection and diagnosis, Doublet et al. (2017), on traffic flow freeway ramp metering, Abouaïssa et al. (2017), on flexible joint manipulator, Agee et al. (2015). The following paragraph, recalls the fundamental principles of model-free setting which is used, in this paper, for supply chain control application.

Classically, many control algorithms rely on "control models" of the system to be controlled, following physical considerations and accurate enough in order to achieve some dynamic performances of the feedback control. In model-free control, there is no need of physical model but the control designer impresses a purely numerical model which involves very few parameters that are estimated thanks to the algebraic methods, online during operation of the system. The feedback control law is built and tuned by the numerical model and is thus updated at each sample time.

2.1 Model-free principle

Assume that the model of the studied system is unknown and consider a system $\mathfrak{S}$, which is assumed to be single-input single-output (SISO) for simplicity's sake. Replace the unknown "global" model by a "phenomenological" model

$$y^{(\nu)}(t) = F(t) + \alpha u_{mfc}(t) \qquad (1)$$

which is

- valid during a short time lapse,
- said to be *ultra-local*,

where

- $y^{(\nu)}(t)$ and $u_{mfc}(t)$ are the derivative of order $\nu \geq 1$ of the controlled output $y(t)$ and input, respectively. ν may always be chosen 1 or 2 (see Fliess and Join (2013)).
- $\alpha \in \mathbb{R}$, is chosen such that $y^{(\nu)}(t)$ and $\alpha u_{mfc}(t)$ are of the same magnitude.
- F is estimated by an algebraic technique (Fliess and Join (2013)). Using values of the output y and the input $u_{mfc}(t)$ at any time sample one can estimate F. The time-varying quantity F which is continuously updated, subsumes not only the un-modeled dynamics, but also all the unknown signals (disturbances, noise).

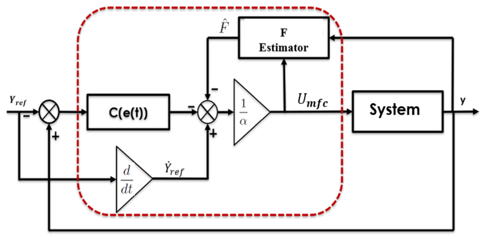

Fig. 1. Model-Free Control Principle

Assuming the estimation of $F(t)$ in Fig. 1, the control law is designed as:

$$u_{mfc} = \frac{1}{\alpha}\left(-F(t) + y_{ref}^{(\nu)}(t) + C(e(t))\right) \qquad (2)$$

with $e(t) = y(t) - y_{ref}(t)$ and $C(e(t))$ is a function of the error. It is chosen such as $y^{\nu} + C(e(t))$ is stable, in the following form for example:

$$C(e(t)) = K_P e(t) + K_I \int e(t) + K_D \frac{de(t)}{dt} \qquad (3)$$

where K_P, K_I and K_D are tuning gains.

2.2 Estimation $\hat{F}$ of F

Under a weak integrability condition, F in (1) may be "well" approximated by a piecewise constant function Φ in a short interval $[t - \sigma, t]$. Let us summarize one of the techniques borrowed from Fliess and Sira-Ramirez (2008).

Rewrite Equation (1) with $\nu = 1$ in operational domain, Yosida (1984):

$$sY(s) - y(0) = \frac{\Phi}{s} + \alpha U_{mfc}(s) \tag{4}$$

where Φ is a constant and $C(e(t)) = K_P e(t)$. To get rid of the initial condition $y(0)$, multiply both sides on the left by $\frac{d}{ds}$.

$$\frac{\Phi}{s^2} = -Y(s) - s\frac{dY(s)}{ds} + \alpha\frac{dU_{mfc}(s)}{ds} \tag{5}$$

For noise attenuation, multiply both sides on the left by s^{-2}, which is equivalent, in time domain to a low-pass filter.

It yields in time domain the real time estimate, thanks to the equivalence between $\frac{d^k}{ds^k}$, $k \geq 1$ and the multiplication by $(-t)^k$ and we use the Cauchy formulae to convert multiple integrals to a single integral.

$$\hat{F} = -\frac{6}{\tau^3}\int_{t-\tau}^{t}\left[(\tau - 2\sigma)y(\sigma) + \alpha\sigma(\tau - \sigma)u_{mfc}(\sigma)\right]d\sigma \tag{6}$$

where $\tau > 0$ might be quite small, Fliess and Join (2021). Notice that in practice, this integral may be replaced by classic digital filter.

3. SUPPLY CHAIN DYNAMIC MODEL

This section presents the simulated supply chain model and the control strategies included in the comparisons.

3.1 Supply chain dynamic model

A supply chain can be seen as a set of n nodes (producer or non-producer) ($n \in \mathbb{N}$) with inventory levels N_i, $i = 1, .., n$ and production or incoming flow rates λ_i.

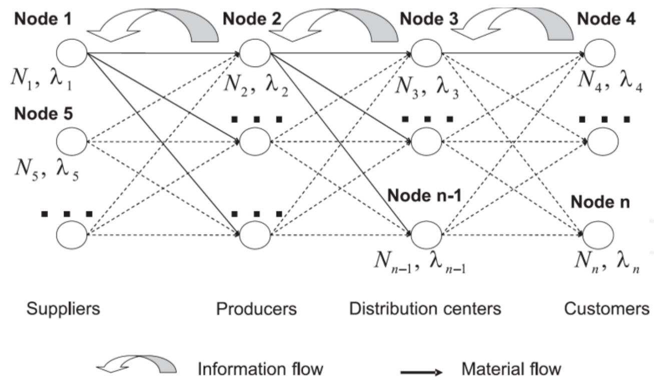

Fig. 2. A general representation of a supply chain system, Rodríguez-Angeles et al. (2009)

For a producer node, the incoming rate varies accordingly to production policies and process dynamical behavior. Change in the production rate involves several activities that require an adaptation time T_i. Then its dynamics can be represented by:

$$\frac{d\lambda_i}{dt} = \frac{1}{T_i}\left(W_i - \lambda_i\right) \tag{7}$$

where W_i denotes the control action that varies the production rate λ_i in a producer node. Contrarily, in a non-producer node, λ_i corresponds only to the materials that are received from its suppliers. By applying macroscopic mass balance with analogy to traffic theory so that the mass variation $\frac{dN_i}{dt}$ is equal to the difference between inflow λ_i and outflow material (the shipped product) y_i, Rodríguez-Angeles et al. (2009). This model includes some information flows, particularly orders among downstream and upstream nodes. Because in some situations, information flow have to be taken into account, either to strive for performance improvement or because not all orders are fulfilled immediately, then an unattended orders balance appears following (8).

$$\frac{dO_i(t)}{dt} = \delta_i - y_i(t) \tag{8}$$

where y_i represents the shipped materials (attended orders) and $\delta_i = \sum_{j=1}^{r} F_{i,j}\lambda_j$ the total product demanding rate (all orders requiring product or material from the *ith*-node with product ratios $F_{i,j}$). When an order is put in node i, it will take an interval of time τ_i (time delay parameter) until it is attended. Following traffic flow theory, a shipping function $y_i \geq 0$ that depends on the inventory level and the accumulated orders is introduced:

$$y_i(t) = \frac{N_i}{N_i + 1}\frac{O_i}{\tau_i} \tag{9}$$

For simplicity sake, in case of producer nodes, it is assumed that upstream nodes deliver the demanded product or material as soon as it is required for production.

- For producer node, the dynamic model is given by (7) and (10):
$$\frac{dN_i(t)}{dt} = \lambda_i(t) - y_i(t) \tag{10}$$
where, $y_i(t) = \delta_i = \sum_{j=1}^{r} F_{i,j}\lambda_j$.
- For non-producer node, the dynamic model is given by (10), (8) and (9).

3.2 Classical controllers of supply chains

A bounded and smooth controllers (CB) proposed in Rodríguez-Angeles et al. (2009), for producers and non-producers nodes are given by (11).

$$W_i = \lambda_i = \lambda_{i,max}\left(2 - \frac{1}{1 + e^{-\alpha_i(N_i - N_{c,i})}} - \frac{1}{1 + e^{-\alpha_i N_{c,i}}}\right) \tag{11}$$

with $N_{c,i} = N_{d,i} + K_p e(t) + K_i \int e(t)$.

The PI law in each node is given by:

$$u_{i,PI} = K_{P_i'} e(t) + K_{I_i'} \int e(t) \tag{12}$$

where $e(t) = (N_{d,i} - N_i)$. $N_{d,i}$ is the desired value of N_i in node i and coefficients K_i and K_p are given in Rodríguez-Angeles et al. (2009).

3.3 Application of MFC (i-P) to supply chain

Model-free control and its corresponding intelligent Proportional controller (*i-P*) derived from (1) and (2) applied to supply chain leads to:

$$\dot{N}_i = \hat{F}_i + \alpha_i U_i \tag{13}$$

and:

$$W_i(t) = \lambda_i(t) = -\frac{\hat{F}_i - \dot{N}_{d,i} + K_{P_i}(N_i - N_{d,i})}{\alpha_i} \tag{14}$$

The physical constraints are given by (15):

$$\begin{cases} N_{i,min} \leq N_i \leq N_{i,max} \\ 0 \leq \lambda_i \leq \lambda_{i,max} \\ 0 \leq W_i \leq \lambda_{i,max} \end{cases} \tag{15}$$

4. APPLICATION AND RESULTS

In this section, we consider a petrochemical supply chain system composed of producer and non-producer nodes. Our control objectives are to maintain the inventory level N_i at a desired level in each node $N_{d,i}$ and to satisfy the demands at the distribution nodes under the physical constraints give by (15).

4.1 A petrochemical case study

Our control algorithm was applied to a case study borrowed from (Rodríguez-Angeles et al. (2009)) and depicted in Fig. 3. The studied supply chain represents an extraction, blending and distribution system for crude oil. Numbers in the left side identifies the nodes. Two types of products (A and B) are extracted in nodes 1 and 2 respectively and are mixed in node 3. The output of blending (mixed oil) is stored in a warehouse (node 4) where it awaits shipments to distribution in nodes 5 and 6.

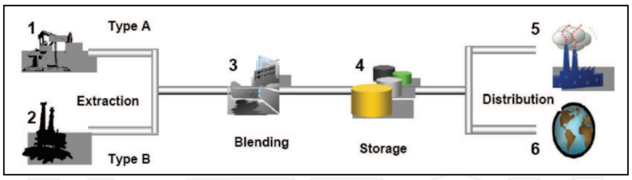

Fig. 3. Extraction, blending and distribution system for crude oil

The different values of desired inventory level $N_{d,i}$ [2] and their initials values $N_{0,i}$; K_{P_i} and α_i (in $i-P$ controllers); and $K_{P'_i}$ and $K_{I'_i}$ for PI are given in table 1.

Table 1. Numerical values

Nodes	$N_{0,i}$	$N_{d,i}$	K_{P_i}	α_i	$K_{P'_i}$	$K_{I'_i}$
Node 1	9000	8000	0.75	10	70	0.002
Node 2	7000	6000	0.78	10	60	0.025
Node 3	4000	2000	0.9	100	1000	0.2
Node 4	4000	3000	1.35	100	20	3
Node 5	2000	1500	1.95	100	15	2
Node 6	2000	1000	0.95	100	20	3

The adaptation time in producer nodes is $T_i = 2$ (h), the time delay parameter for non-producer nodes is $\tau_i = 0.25$ (h) and customers demands in nodes 5 and 6 are $\delta_5 = 150$ (Mt/h) and $\delta_6 = 200$ (Mt/h), respectively.

4.2 Simulation results

Fig. 4 shows that, in node 6 all controllers converge to the desired value $N_{d,6} = 1000$ (Mt). It can be seen that the model-free control reaches more quickly the desired level after around 25h with a maximal production rate $\lambda_{6,MFC} = 235$ (Mt/h) against 38h and 50h obtained by the other controllers with $\lambda_{6,PI} = \lambda_{6,CB} = 250$ (Mt/h). Notice that around the intervals $t = [5-14]$h for MFC, $t = [5-40]$h for PI and $t = [5-28]$h for CB, we observe inventory deficit that occurs when the difference between the actual inventory level N_i and the desired one $N_{d,i}$ is negative: $I_d = N_i - N_{d,i} < 0$. With the MFC, $I_d = 179$ (Mt) while in PI and CB controllers they are equal to 363 (Mt) and 442 (Mt) respectively. This means in practice that the MFC strategy is less likely to reach a critical inventory level. The same results and observations are obtained for the other non-producer nodes (distribution node 5 and storage node 4).

[2] The different values are expressed in Mega-tonnes (Mt) for the levels; T_i and τ_i in hours [h] and λ_i in (Mt/h).

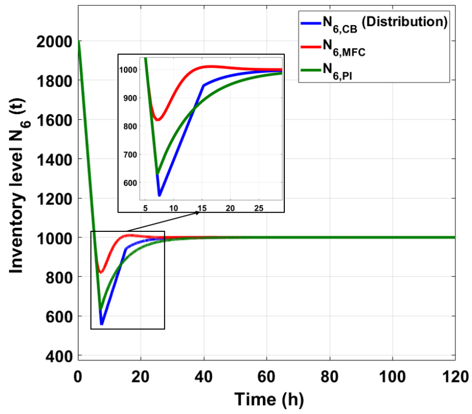

Fig. 4. Inventory level N_6 at node 6

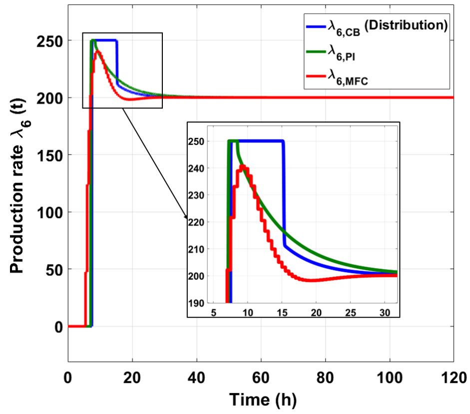

Fig. 5. Production rate λ_6 at node 6

Consider now that in dynamic behavior of non-producer nodes, an unattended orders balance (8) appears (see for example with MFC, Fig. 6) due to the interval of time (τ_i) until orders are delivered (see λ_6 in Fig. 5).

The unattended orders in non-producer nodes 4, 5 and 6 generate an overshoot on the inventory level $N_3(t)$ of producer node 3 which is the direct upstream entity (see Fig. 7).

In Fig. 7, we observe that the MFC presents a lower overshoot. One can notice that the fluctuations observed in the producers nodes (see Fig. 7 and Fig. 8) are due to the adaptation time T_i of the factories and τ_i during transients at demands and control actions such that bullwhip phenomena can arise.

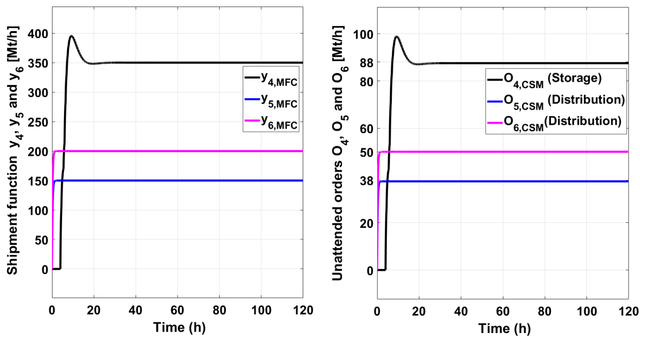

Fig. 6. Shipment orders y_i and O_i in non-producers nodes

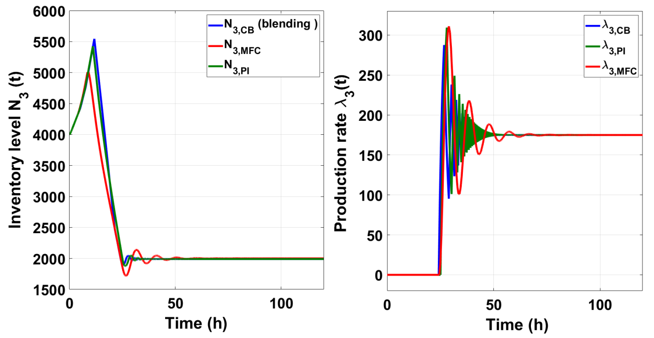

Fig. 7. Inventory level N_3 at node 3

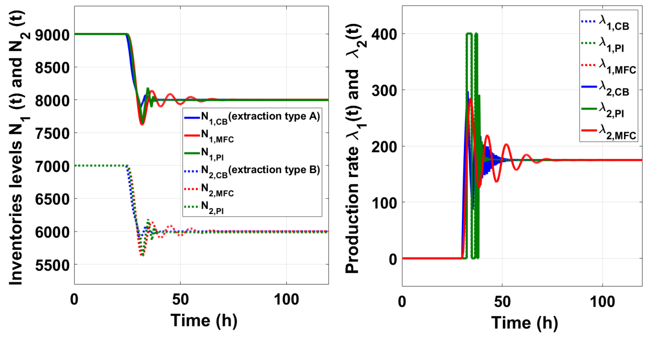

Fig. 8. Inventory levels N_1, N_2 and production rates λ_1, λ_2 at nodes 1 and 2

The presented results show that our control objectives are reached, in Fig. 9, we test the situation where the demand is corrupted by a noise and when it abruptly change at 100h (demand decreases to 50 (Mt/h) for example).

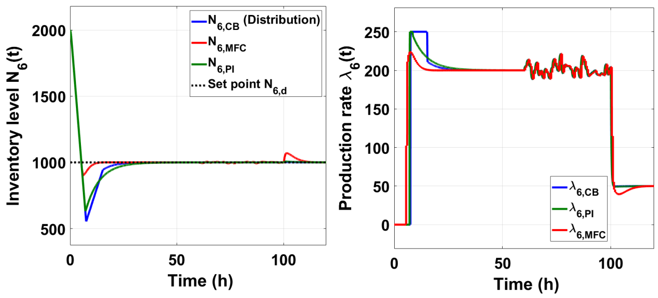

Fig. 9. Inventory level N_6 and production rate λ_6

One can see that the noise is somehow filtered in the inventory level and the control objectives are insured despite the demands abrupt changes.

The following section considers the case where dynamic model of the supply chain is known with unknown parameters. We propose an online parameters estimation using an algebraic estimation technique introduce few years ago by, Fliess and Sira-Ramírez (2003).

5. REAL-TIME ESTIMATION VIA ALGEBRAIC TECHNIQUES

This approach which exhibits good robustness properties with respect to a large variety of additive perturbations is based on the following mathematical tools: module theory, differential algebra and operational calculus (see Fliess et al. (2008), Fliess and Sira-Ramírez (2003)). Their algebraic nature permits to derive exact non-asymptotic formulae for obtaining the unknown quantities in real time. There is no need to know the statistical properties of the corrupting noises and the initials values, Fliess and Sira-Ramirez (2008), Tian et al. (2008). In order to estimate the parameters T_i in (7), one can proceed by:

Step 1: Applying Laplace transforms (Spiegel (1965)) to (7), we obtain (16):

$$\lambda_i(s) - \lambda_i(0) = \frac{1}{T_i}\left(W_i(s) - \lambda_i(s)\right) \tag{16}$$

Step 2: Taking the first derivative with respect to s permits to ignore the initial condition $\lambda_i(0)$:

$$\lambda_i(s) + s\frac{d\lambda_i(s)}{ds} = \frac{1}{T_i}\left(\frac{dW_i(s)}{ds} - \frac{d\lambda_i(s)}{ds}\right) \tag{17}$$

Multiply both sides of (17) by s^{-2} for avoiding derivations with respect to time:

$$s^{-2}\lambda_i(s) + s^{-1}\frac{d\lambda_i(s)}{ds} = \frac{1}{T_i}\left(s^{-2}\frac{dW_i(s)}{ds} - s^{-2}\frac{d\lambda_i(s)}{ds}\right) \tag{18}$$

Step 3: A time-domain representation of $T_{i,e}$ is given by (19):

$$T_{i,e} = \frac{\int^{(2)} t\lambda_i(t) - \int^{(2)} tW_i(t)}{\int^{(2)} \lambda_i(t) - \int t\lambda_i(t)} \tag{19}$$

And $\tau_{i,e}$ is obtained using the first equation of system (10) where y_i is given by (9). The same steps like $T_{i,e}$ are applied, we then have:

$$\tau_{i,e} = \frac{\int^{(2)} tC_i(t)}{\int^{(2)} t\lambda_i(t) - \int tN_i + \int^{(2)} N_i} \tag{20}$$

where, $C_i(t) = \frac{N_i}{N_i+1}O_i$.

For the simulations we consider $T_3 = 2$ (h) and $\tau_5 = 0.25$ (h).

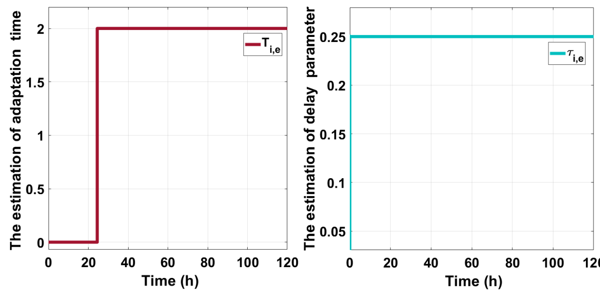

Fig. 10. The estimations of $T_{i,e}$ and $\tau_{i,e}$

In Fig. 10, we observe that the estimation of parameters is fast and the exact value of T_3 and τ_5 are obtained almost instantaneously. $T_{3,e} = 0$ about $t = 25$ (h) due to the fact that λ_3 begin at this time (it is the same in other producers nodes).

6. CONCLUSION

Based on a real example of a petrochemical supply chain, the paper provides a robust and easy technique to tackle the supply chain management which avoids the use of an exact model. It demonstrates that model-free control is an effective and a successful approach for dynamic inventory management in supply chains systems. It discusses results of dynamic analysis and a comparative studies that demonstrate the relevance of the control approach and show that this strategy can significantly lower the impact of demand variability. In addition, the paper considers the case of a supply chain described by an incomplete (or restricted) model and introduces a deterministic approach based on algebraic methods for parametric estimation. Such method will be used in the future for demand prediction and forecasting. In addition further works will provide a general framework for decentralized supply chain management where model-free control plays an important role.

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