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1 Preference in Abstract Argumentation

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ABSTRACT. Preference is a key concept in argumentation to represent the comparative strength of arguments. In abstract argumentation, it is represented by an ordinal comparative, or by a numerical function. In this chapter, we study the role of comparative preference in abstract argumentation, and numerical preferences are studied in an accompanying chapter. This chapter consists of two parts. In the first part, we survey four reductions discussed in the literature to provide semantics to preference-based argumentation frameworks, and we present ten principles for such semantics. Some of these principles have been mentioned in the literature before, and some of them are new. We provide a complete analysis for the four Dung semantics, based on these four reductions and the ten principles. In the second part of the chapter, we give an outlook of the various open research challenges concerning preference in abstract argumentation. We discuss alternative semantics not based on reductions, principles in the context of symmetric attack, the relation to structured argumentation, and the dynamics of preference and argumentation.

1 Introduction

In general, as witnessed by the first volume of the Handbook of Formal Argumentation [Baroni et al., 2018], research in formal argumentation is evenly balanced between studies of abstract argumentation, and studies in structured argumentation. At the abstract level, there are dialogue games, algorithms and a principle-based analysis of the various semantics [Baroni and Giacomin, 2007; van der Torre and Vesic, 2018], and at the structured level there are argument schemes, dialogues, algorithms, and rationality postulates.

However, the balance changes when we consider the concept of preference among arguments. Preferences are used in argumentation to represent the comparative strength of arguments, a natural and commonly adopted concept in argumentation. Therefore, in structured argumentation, preference plays a central role in the formal theories and results. Maybe surprisingly, there exist very few formal results and studies about the role of preference in abstract argumentation. This handbook addresses this gap in two chapters. In this chapter, we study the role of comparative preference in abstract argumentation, while the numerical preferences are studied in an accompanying chapter. In the first part, we survey and extend the formal study on preference in abstract argumentation, and in the second part we discuss open challenges.
First, most of the formal results that involve preference in abstract argumentation are concerned with different ways to define the semantics of preference-based argumentation frameworks, based on different relations between the concept of attack and the concept of preference at the abstract level. In the traditional approach [Amgoud and Cayrol, 2002], an argument defeats another one if it attacks it and the attacked argument is not preferred to the attacker. Moreover, Amgoud and Vesic [2014] propose an alternative reduction, in which the attack may be reversed, and Kaci et al. [2018] propose two more alternatives. This raises the question how we can choose among these four reductions for a particular application. Moreover, it raises the question whether there are more alternatives, and if so, how we can guide the search for possibly alternative semantics for preference-based argumentation.

In addition, these different semantics are often analyzed using a principle-based approach, analogous to the principle-based analysis of the semantics of argumentation frameworks. Amgoud and Vesic [2014] study so-called Conflict-freeness and Generalisation. Kaci et al. [2018] introduce Extension Growth, which says that if we add preferences, then we can infer more, and Extension Selection, where the intersection of the extensions grows because there are fewer extensions, but the extensions do not become larger. However, they do not provide a complete analysis. Moreover, compared to the number of principles studied in abstract argumentation in general, the number of principles investigated for preference-based argumentation is quite small. In this chapter, we survey and extend this principle-based analysis of preference-based argumentation frameworks.

Second, we give an overview of future research directions concerning the formal study of the role of preference in abstract argumentation. More precisely, we question the relation between attack and preference by considering alternative semantics not based on reductions, and then we consider the representation of preference-based argumentation frameworks. In particular, we consider principles which hold only for symmetric attack, which constitutes a particularly promising fragment of preference-based argumentation. Third, we consider the relation to structured argumentation, and finally we consider the dynamics of preference and argumentation.

From these challenges, the relation with structured argumentation is subject of debate. Roughly, whereas we handle preference at the abstract level of Dung’s Argumentation Framework (AF), assuming a preference order between arguments, some researchers are skeptical about this approach, on the grounds that the assumptions made at the abstract level may not hold when considering preferences at the structured level [Modgil and Prakken, 2012]. In the opinion of these authors, although the abstract approach is very popular in some research groups, it lies on a very shaky ground. In particular, it involves two binary relations on the set of arguments, where usually none of them has any restrictions, and the interaction between them is not restricted either. As arguments are treated in an absolutely abstract way, the proponents of the
second position claim, it is no wonder that strange things may happen. We do not disagree with this observation, which in particular impacts on the study of dynamics of argumentation. However, we believe this observation does not discredit the use of a principle-based analysis. Moreover, whereas most structured approaches use the traditional reduction, in our opinion the alternative reductions can be adopted in structured argumentation as well.

The layout of this chapter is as follows. In Section 2, we introduce the semantics of preference-based argumentation frameworks. In Section 3, we introduce our ten principles, and we verify whether the semantics satisfy the ten principles. Section 4 discusses various open challenges concerning preference in abstract argumentation.

2 Preference-based abstract argumentation

In this section, we introduce the semantics of preference-based argumentation frameworks, based on a Dung semantics and one out of four reductions.

2.1 Dung’s argumentation theory

Dung introduced an approach where the acceptance of an argument depends only on the defeat relation among arguments and the chosen argumentation semantics, but not on the internal structure of the arguments [Dung, 1995a]. Dung [1995a] refers to an “attack” relation, but in preference-based argumentation “attack” is used for something else, see Definition 2.4 below.

Definition 2.1 (Argumentation framework [Dung, 1995a]) An argumentation framework (AF) is a tuple \( \langle A, \text{Def} \rangle \) where \( A \) is a set of arguments and \( \text{Def} \subseteq A \times A \) is a binary defeat relation.

In this chapter we suppose that the set of arguments of each argumentation framework is finite. Given \( a, b \in A \), \( (a, b) \in \text{Def} \) stands for \( a \) defeats \( b \). The outcome of an argumentation framework is a set of sets of arguments, called extensions and denoted by \( E(A, \text{Def}) \), that are robust against defeats. The extensions rely on two conditions namely conflict-freeness and defense.

Definition 2.2 (Conflict-freeness & Defense [Dung, 1995a]) Let \( \langle A, \text{Def} \rangle \) be an AF.

- \( \mathcal{A} \subseteq A \) is conflict-free if there are no \( a, b \in \mathcal{A} \) such that \( (a, b) \in \text{Def} \).
- \( \mathcal{A} \subseteq A \) defends \( c \) if \( \forall b \in A \) with \( (b, c) \in \text{Def} \), \( \exists a \in \mathcal{A} \) such that \( (a, b) \in \text{Def} \).

We distinguish several definitions of extension, each corresponding to an acceptability semantics that formally rules the argument evaluation process.

Definition 2.3 (Acceptability semantics [Dung, 1995a]) Let \( \langle A, \text{Def} \rangle \) be an AF.

- \( S \subseteq A \) is admissible iff it is conflict-free and defends all its elements.
• A conflict-free $S \subseteq A$ is a complete extension iff $S = \{ A \mid S \text{ defends } A \}$.
• $S \subseteq A$ is the grounded extension iff it is the smallest (for set inclusion) complete extension.
• $S \subseteq A$ is a preferred extension iff it is a largest (for set inclusion) complete extension.
• $S \subseteq A$ is a stable extension iff it is a preferred extension that defeats all arguments in $A \setminus S$.

2.2 Preference-based argumentation frameworks

A preference-based argumentation framework may be seen as an instance of Dung’s framework. It is based on a binary attack relation between arguments and a preference relation over the set of arguments.

Definition 2.4 (Preference-based argumentation framework) [Simari and Loui, 1992; Amgoud and Cayrol, 2002] A Preference-based Argumentation Framework (PAF) is a 3-tuple $\langle A, Att, \succeq \rangle$ where $A$ is a set of arguments, $Att$ is a binary attack relation $\subseteq A \times A$ and $\succeq$ is a second binary relation over $A$, called preference relation. Moreover, we write $a \succ b$ for $a \succeq b$ and not $b \succeq a$.

Given $a, b \in A$, $(a, b) \in Att$ stands for $a$ attacks $b$. By convention it is often assumed that preferences are transitive, or that the preference relation $\succ$ is an order (irreflexive and transitive). However, it is not always the case that preference satisfies transitivity in real world scenarios. Moreover, this is an issue orthogonal to the discussion in this paper. In this chapter, we do not impose the transitivity of preferences. Only Theorem 4.9 depends on the validity of transitivity for the preference relation.

To compute the extensions of a preference-based argumentation framework $\langle A, Att, \succeq \rangle$ the latter can be reduced to a Dung’s AF $\langle A, Def \rangle$, as we will see in Section 2.3. The extensions of a preference-based argumentation framework, denoted by $E(\langle A, Att, \succeq \rangle)$, are simply the extensions of the argumentation framework it represents.

We end this subsection with an observation further discussed in section 4.1. A preference-based argumentation framework may also be seen as an extension instead of an instance of an argumentation framework, in the sense that if the preference relation is the universal relation, such that every argument is equally preferred, the attack and the defeat relation coincide. In this sense, we can also define semantics for preference-based argumentation frameworks which are not based on a reduction.

2.3 Reductions: from PAF to AF

In this section, we present different ways to reduce a PAF into Dung’s AF and we illustrate them through a running example.
Reduction 1 [Amgoud and Cayrol, 2002] has been used widely in preference-based argumentation. The basic idea is that an attack succeeds only when the attacked argument is not preferred to the attacker.

**Definition 2.5 (Reduction 1 [Amgoud and Cayrol, 2002])** Let \( \langle A, \text{Att}, \succeq \rangle \) be a preference-based argumentation framework and \( \langle A, \text{Def} \rangle \) be the AF it represents. Then, \( \forall a, b \in A \),

\[(a, b) \in \text{Def} \text{ iff } (a, b) \in \text{Att}, b \not\succ a.\]

This reduction has been criticised by Amgoud and Vesic [2009; 2010] as it may lead to extensions that are not conflict-free. The problem occurs when there is an attack from an argument to a preferred argument. This attack is called critical by Amgoud and Vesic [2010].

**Example 2.6 (Reduction 1)** Let \( \langle A, \text{Att}, \succeq \rangle \) with \( A = \{a, b\} \), \( \text{Att} = \{(a, b)\} \) and \( b \succeq a \). We have \( \text{Def} = \emptyset \). Both \( a \) and \( b \) are accepted using any semantics although they are conflicting w.r.t. \( \text{Att} \).

Amgoud and Vesic [2010] propose to repair the argumentation framework to avoid this drawback. They extend Reduction 1 by enforcing a defeat from an argument to another when the former is preferred but attacked by the latter. This is Reduction 2.

**Definition 2.7 (Reduction 2 [Amgoud and Vesic, 2010])** Let \( \langle A, \text{Att}, \succeq \rangle \) be a preference-based argumentation framework and \( \langle A, \text{Def} \rangle \) be the AF it represents. Then, \( \forall a, b \in A \),

\[(a, b) \in \text{Def} \text{ iff } (a, b) \in \text{Att}, b \not\succ a, \text{ or } (b, a) \in \text{Att}, (a, b) \not\in \text{Att}, a \succ b.\]

**Example 2.8 (Example 2.6 continued – Reduction 2)** We have \( (b, a) \in \text{Def} \). \( b \) is accepted.

Reduction 2 solves the shortcoming of Reduction 1. Kaci et al. [2018] argue that nevertheless it is based on an implicit strong constraint. That is, an argument never succeeds to attack a preferred argument. This view gives a power to preferred arguments which goes against the idea underlying argumentation. Kaci et al. consider a parent who refuses that his child watches TV in the evening during the week because he has courses the next day. However, the child says that his courses have been cancelled. Then to maintain the refusal to watch TV the parent should provide another argument attacking his child’s argument. The idea is: if an argument is attacked by a less preferred argument (critical attack) then the former should defend itself against its attacker. They formalize this idea in Reduction 3.
Figure 1. The defeat relation is visualised using thick lines, and if arguments are accepted in grounded semantics (and thus in extensions of all semantics considered in this paper) they are visualised with a thick circle. From left to right: the original argumentation framework, the result after applying Reduction 1, Reduction 2, Reduction 3 and Reduction 4.

Definition 2.9 (Reduction 3 [Kaci et al., 2018])
Let $(\mathcal{A}, \text{Att}, \succeq)$ be a preference-based argumentation framework and $(\mathcal{A}, \text{Def})$ be the AF it represents. $\forall a, b \in \mathcal{A}$: $(a, b) \in \text{Def}$ iff

- $(a, b) \in \text{Att}, b \not\succeq a$, or
- $(a, b) \in \text{Att}, (b, a) \not\in \text{Att}$

Example 2.10 (Example 2.6 continued – Reduction 3) We have $(a, b) \in \text{Def}$. $a$ is accepted.

Reduction 3 in turn can also be criticized arguing that it would not be natural to make successful an attack from a less preferred argument. Reduction 4 below mixes Reduction 2 and 3.

Definition 2.11 (Reduction 4 [Kaci et al., 2018])
Let $(\mathcal{A}, \text{Att}, \succeq)$ be a preference-based argumentation framework and $(\mathcal{A}, \text{Def})$ be the AF it represents. $\forall a, b \in \mathcal{A}$: $(a, b) \in \text{Def}$ iff

- $(a, b) \in \text{Att}, b \not\succeq a$ or
- $(b, a) \in \text{Att}, (a, b) \not\in \text{Att}, a \succ b$ or
- $(a, b) \in \text{Att}, (b, a) \not\in \text{Att}$

Example 2.12 (Example 2.6 continued – Reduction 4) We have $(a, b) \in \text{Def}$ and $(b, a) \in \text{Def}$. The grounded extension is empty, complete extensions are $\{\}$, $\{a\}$, $\{b\}$, and there are two preferred/stable extensions $\{a\}$ and $\{b\}$.

Figure 1 illustrates the differences between the four reductions.
3 Principle-based comparison and analysis

We now define the status of an argument. Note that we want each argument to have one and exactly one status. This helps us to simplify the presentation in the rest of the chapter.

**Definition 3.1 (Status of an argument)** Let $\mathcal{F} = (\mathcal{A}, \text{Att}, \succeq)$ be a PAF. If the set of extensions is empty, all the arguments are declared to be rejected. Otherwise, we say that an argument is

- skeptically accepted, if it belongs to all extensions;
- credulously accepted, if it is not skeptically accepted and it belongs to at least one extension;
- rejected, if it does not belong to any extension.

We write $\text{Status}(a, \mathcal{F}) = \text{sk}$ (resp. $\text{cr}$, $\text{rej}$) when $a$ is skeptically accepted (resp. credulously accepted, rejected). We define the order $\succeq$ on the set of statuses as expected: $\text{sk} > \text{cr} > \text{rej}$. We denote the set of skeptically accepted (resp. credulously accepted, rejected) arguments of a PAF by $\text{Sk}(\mathcal{A}, \text{Att}, \succeq)$ (resp. $\text{Cr}(\mathcal{A}, \text{Att}, \succeq)$, $\text{Rej}(\mathcal{A}, \text{Att}, \succeq)$).

We define the path following the standard definition in the literature, in which we can travel over the directed arcs in both directions.

**Definition 3.2 (Path)** For two arguments $a, b \in \mathcal{A}$, we say that there is a path between $a$ and $b$ if there exist $a_1, \ldots, a_n \in \mathcal{A}$ such that $(a, a_1), (a_1, a_2), \ldots, (a_n, b) \in \text{Att} \cup \text{Att}^{-1}$.

3.1 Principles

We now introduce the principles which we use then for our principle-based analysis of preference-based argumentation frameworks. Recall that we do not suppose that the preference relation is transitive. Also, we do not suppose that transitive closure is added in any case unless explicitly stated. The first principle says that an extension cannot contain arguments that attack each other.

**Principle 1 (Conflict-freeness)**

If $(a, b) \in \text{Att}$ then $\not\exists E \in \mathcal{E}(\mathcal{F})$ s.t. $\{a, b\} \in E$.

The second principle says that adding preferences helps to keep only some of the extensions.

**Principle 2 (Preference selects extensions 1)**

$\mathcal{E}(\mathcal{A}, \text{Att}, \succeq \cup \succeq') \subseteq \mathcal{E}(\mathcal{A}, \text{Att}, \succeq)$

The third principle is a special case of the second principle.
Principle 3 (Preference selects extensions 2)
\[ \mathcal{E}(\mathcal{A}, \mathcal{A}, \succ) \subseteq \mathcal{E}(\mathcal{A}, \mathcal{A}, \emptyset) \]

The fourth principle says, roughly speaking, that extensions grow when preferences are added.

Principle 4 (Extension refinement)
\[ \forall E' \in \mathcal{E}(\mathcal{A}, \mathcal{A}, \succ \cup \succ'), \exists E \in \mathcal{E}(\mathcal{A}, \mathcal{A}, \succ) | E \subseteq E' \]

The fifth principle says that each skeptically accepted argument stays skeptically accepted when preferences are added.

Principle 5 (Extension growth)
\[ \text{Sk}(\mathcal{A}, \mathcal{A}, \succ) \subseteq \text{Sk}(\mathcal{A}, \mathcal{A}, \succ \cup \succ') \]

The sixth principle says that adding preferences cannot increase the number of extensions.

Principle 6 (Number of extensions)
\[ | \mathcal{E}(\mathcal{A}, \mathcal{A}, \succ \cup \succ') | \leq | \mathcal{E}(\mathcal{A}, \mathcal{A}, \succ) | \]

The seventh principle says that adding a preference \( a \succ b \) cannot worsen the status of argument \( a \).

Principle 7 (Status conservation)
\[ \text{Status}(a, (\mathcal{A}, \mathcal{A}, \succ \cup \{(a, b)\})) \geq \text{Status}(a, (\mathcal{A}, \mathcal{A}, \succ)) \]

The eighth principle says that if an argument is strictly preferred to all the other arguments, it is not rejected.

Principle 8 (Preference-based immunity)
If \((a, a) \notin \mathcal{A} \text{ and } \forall b \in \mathcal{A} \setminus \{a\}, a \succ b\) then \( \text{Status}(a, (\mathcal{A}, \mathcal{A}, \succ)) \neq \text{rej} \)

The ninth principle says that if there is no path between \( a \) and \( b \), then adding preferences between them will not change anything.

Principle 9 (Path preference influence 1) If there is no path between \( a \) and \( b \) then:
\[ \mathcal{E}(\mathcal{A}, \mathcal{A}, \succ) = \mathcal{E}(\mathcal{A}, \mathcal{A}, \succ \cup \{(a, b)\}) \]

The last principle is a variant of the ninth one.
Principle 10 (Path preference influence 2) If \((a, b) \notin \text{Att}\) and \((b, a) \notin \text{Att}\), then:

\[
\mathcal{E}(\mathcal{A}, \text{Att}, \succeq) = \mathcal{E}(\mathcal{A}, \text{Att}, \succeq \cup \{(a, b)\})
\]

The first six principles have already been presented in the literature [Kaci et al., 2018], while the other principles are new. It is immediate to see that P2 implies P3, P4 and P6. Also, for semantics that always return at least one extension, P2 implies P5.

Observe that P7 can be equivalently stated as

\[
\text{Status}(a, (\mathcal{A}, \text{Att}, \succeq \cup \{(a, b) \mid b \in B\})) \geq \text{Status}(a, (\mathcal{A}, \text{Att}, \succeq))
\]

It is immediate to see that the previous formulation implies P7. Conversely, if a semantics satisfies P7, we can show that it satisfies the previous equation by successively applying P7 several times (to be precise, \(|B|\) times).

We can also note that under those semantics that always return at least one extension, P4 implies P5.

**Proposition 3.3** Let \(\mathcal{E}\) be a semantics that always returns at least one extension. If \(\mathcal{E}\) satisfies P4, then it satisfies P5.

**Proof.** Let \(a \in \text{Sk}(\mathcal{A}, \text{Att}, \succeq)\) be an arbitrary skeptically accepted argument of \((\mathcal{A}, \text{Att}, \succeq)\). Let \(E' \in \mathcal{E}(\mathcal{A}, \text{Att}, \succeq \cup \succeq')\). Since \(\mathcal{E}\) always returns at least one extension, there is at least one such \(E'\). Let \(E \in \mathcal{E}(\mathcal{A}, \text{Att}, \succeq)\) such that \(E \subseteq E'\). (Such a set exists since P4 is satisfied.) Hence, \(a \in E\). Consequently, \(a \in E'\). Since \(E'\) was arbitrary, we conclude that \(a\) belongs to every extension of \((\mathcal{A}, \text{Att}, \succeq \cup \succeq')\). Furthermore, \((\mathcal{A}, \text{Att}, \succeq \cup \succeq')\) has at least one extension. Hence, \(a \in \text{Sk}(\mathcal{A}, \text{Att}, \succeq \cup \succeq')\). \(\blacksquare\)

We see from their respective definitions that P9 implies P10.

### 3.2 Satisfaction of principles

We now study which principles are satisfied by which reductions. The table summarizing all the results is depicted in Figure 1 on page 17. We start by R1. To see that P1 is not satisfied it is sufficient to consider the graph with only two arguments \(a\) and \(b\), such that \(\text{Att} = \{(a, b)\}\) and \(b \succeq a\). For all four studied semantics, we have a unique extension \(\{a, b\}\).

P3 is not satisfied by any semantics, as shown by the counter-example in figure 2. Since P2 implies P3 and P3 is not satisfied, we conclude that P2 is not satisfied.

P4 and P5 are also violated by all the four semantics. Consider the graph in figure 3. The unique extension of \((\mathcal{A}, \text{Att}, \emptyset)\) is the set \(\{a, c\}\) and the unique extension of \((\mathcal{A}, \text{Att}, \{(b, a)\})\) is the set \(\{a, b\}\). Hence, both P4 and P5 are violated.

P6 is trivially satisfied by grounded semantics. It is violated by complete, preferred and stable semantics, as can be seen from the counter-example in Figure 4.
Figure 2. Example showing that P3 is not satisfied by R1 nor by R2 under any of the four semantics. In the rest of the chapter we follow the same way of depicting the argumentation frameworks in the figures, namely, the original framework on the left, and the framework after applying the reduction on the right. In this, as well as in the following examples, we leave the calculation of the extensions to the reader.

Figure 3. Example showing that P5 is not satisfied by R1 under any of the four semantics

Figure 4. Example showing that P6 is violated by R1 and R2 under complete, preferred and stable semantics. Due to the size of the argumentation framework, in order to keep the chapter within a reasonable page limit, we do not depict its version after applying the reductions.
P7 is satisfied by the four semantics. Namely, Amgoud and Vesic [2020] show that deleting an attack can only improve the status of an argument but never worsen it. Under R1, adding a preference \((a, b)\) will result either in deleting the attack \((b, a)\), if it exists or will induce no change in the argumentation graph. Hence, the status of \(a\) cannot be worsen.

P8 is satisfied under all semantics except stable, since a non-attacked argument is in all extensions. The principle is not satisfied under stable semantics, because there might not be any extensions.

P9 and P10 are satisfied since adding a preference between \(a\) and \(b\) does not influence the graph. Hence, the extensions of the two argumentation graphs coincide trivially, since the graphs coincide.

Let us now study the reduction R2. P1 is satisfied since the reduction preserves conflict-free sets, i.e. for every set of arguments \(S \subseteq A\), \(S\) is conflict-free in the original graph if and only if \(S\) is conflict-free in the graph obtained after applying reduction R2.

Figure 2 shows that P3 is not satisfied by R2. Consequently, P2 is not satisfied neither.

P4 and P5 are not satisfied, as can be seen from the example in Figure 5.

P6 is trivially satisfied by grounded semantics. The other semantics violate this principle, as illustrated by the example in figure 4.

**Proposition 3.4** R2 satisfies P7 under all the studied semantics.

**Proof.** Denote by \(\mathcal{F}\) (resp. \(\mathcal{F}'\)) the graph obtained after applying R2 on \((A, Att, \succeq)\), resp. \((A, Att, \succeq \cup \{(a, b)\})\).

If \((b, a) \notin Att\), \(\mathcal{F}\) and \(\mathcal{F}'\) coincide. If \((a, b) \in Att\) and \((b, a) \in Att\), the proof follows from the fact that removing an attack on \(a\) cannot worsen its status [Amgoud and Vesic, 2020]. In the rest of the proof, we study the case when \((b, a) \in Att\) and \((a, b) \notin Att\).

**Stable semantics.**

Let \(a\) be skeptically accepted in \(\mathcal{F}\). By means of contradiction, suppose that \(a\) is not skeptically accepted in \(\mathcal{F}'\); let \(E\) be an extension of \(\mathcal{F}'\) such that \(a \notin E\). Note that \(E\) is conflict-free in \(\mathcal{F}'\); it also attacks all the arguments in its exterior in \(\mathcal{F}\). Hence, \(E\) is a stable extension of \(\mathcal{F}\). Contradiction.

Let \(a\) be credulously accepted in \(\mathcal{F}\). Let \(E\) be an extension of \(\mathcal{F}\) such that \(a \in \mathcal{F}\). Since \(E\) attacks all arguments in its exterior in \(\mathcal{F}\), \(E\) also attacks
all arguments in its exterior in $F'$. Hence, $E$ is a stable extension of $F'$.
Consequently, $a$ is not rejected in $F'$.

\textbf{Preferred semantics.}

Let $a$ be skeptically accepted in $F$. By means of contradiction, let $E$ be an extension of $F'$ such that $a \notin E$.

Case 1: $b \in E$. $E$ is conflict-free in $F$. It is also admissible in $F$. Thus, there exists $E'$ such that $E \subseteq E'$ and $E'$ is an extension of $F$. Since $b$ attacks $a$, $a \notin E'$. Thus, $a$ is not skeptically accepted in $F$, contradiction.

Case 2: $b \notin E$. Note that $E$ is admissible in $F$. Let $E'$ be such that $E \subseteq E'$ and $E'$ is an extension of $F$. Since $a$ is skeptically accepted, $a \in E'$. Note that $E'$ is admissible in $F'$. Hence, $E$ is not an extension of $F'$, contradiction.

Let $a$ be credulously accepted in $F$. Let $E$ be an extension of $F$ such that $a \in E$. Set $E$ is admissible in $F'$, hence there exists $E'$ such that $E \subseteq E'$ and $E'$ is an extension of $F'$. Thus, $a$ is not rejected in $F'$.

\textbf{Grounded semantics.}

Let $f_C$ be the function such that, for every set of arguments $S$, $f_C(S)$ is the set of arguments defended by $S$. Then, the grounded extension is the least fixed point of $f_C(\emptyset)$ [Dung, 1995b]. Denote the grounded extension of $F$ by $GE$ and the grounded extension of $F'$ by $GE'$. Suppose that $a \in GE$ in $F$.

Let $E_0$ be the set of non-attacked arguments of $F$ and let $E'_0$ be the set of non-attacked arguments of $F'$.

Since $b$ attacks $a$ in $F$, $a \notin E_0$. Since $a \in GE$, $b \notin GE$.

For each $i \in \{0, 1, \ldots\}$ let $E_{i+1} = f_C(E_i)$ and $E'_{i+1} = f_C(E'_i)$. Let us prove by induction on $i$ that $E_i \subseteq E'_i$ for every $i$.

Base: $E_0 \subseteq E'_0$ since $a \notin E$ and $b \notin E$.

Step: Let $E_i \subseteq E'_i$. Let $x$ be an arbitrary argument defended by $E_i$. For each $y$ such that $y$ attacks $x$ in $F$, there exists $z \in E_i$ such that $z$ attacks $y$ in $F$. Note that $z \neq b$, since $b \notin GE$. Hence, since $z \in E'_i$, $x$ is defended by $E'_i$ in $F'$. Thus, $E_i \subseteq E'_i$.

Consequently, $GE \subseteq GE'$. Thus, $a$ is accepted in $F'$.

\textbf{Complete semantics.}

Let $a$ be skeptically accepted in $F$. Skeptical acceptance under complete semantics coincides with acceptance under grounded semantics, and we already showed the property for grounded semantics.

Let $a$ be credulously accepted in $F$. Let $E$ be an extension of $F$ such that $a \in E$. Note that $E$ is admissible in $F'$. Hence, there exists a complete extension $E'$ in $F'$ such that $E \subseteq E'$. This means that $a$ is not rejected in $F'$.

$P_8$ is satisfied by preferred, complete and grounded semantics since a non-attacked argument belongs to all extensions under those semantics. It is not satisfied under stable semantics, since stable extensions might not exist.

$P_9$ and $P_{10}$ are satisfied since the preference between $a$ and $b$ can only influence the attacks between $a$ and $b$. 

$\blacksquare$
Let us now study R3. P1 is satisfied since the reduction preserves conflict-free sets.

P3 is not satisfied by grounded semantics, as shown by the example in figure 6. Consequently, P2 is not satisfied by grounded semantics neither. Figure 7 shows that P2 and P3 are violated by preferred semantics.

**Proposition 3.5** R3 satisfies P2 under complete and stable semantics.

**Proof.** Let \( \mathcal{F} \), resp. \( \mathcal{F}' \) be the argumentation graph obtained after applying R3 on \((\mathcal{A}, \text{Att}, \succeq)\), resp. \((\mathcal{A}, \text{Att}, \succeq \cup \succeq')\).

**Complete semantics.**

Suppose \( E \) is an extension of \( \mathcal{F}' \) and prove it is an extension of \( \mathcal{F} \). Note the \( E \) is conflict-free in \( \mathcal{F} \). Let us show that \( E \) is admissible in \( \mathcal{F} \). Let \( x \in E \) and let \( y \) attack \( x \) in \( \mathcal{F} \). If \( x \) attacks \( y \) in \( \mathcal{F} \), it defends itself. Else, suppose that \( x \) does not attack \( y \) in \( \mathcal{F} \). This means that \( x \) does not attack \( y \) in \( \mathcal{F}' \). Hence, there exists \( z \in E \) such that \( z \) attacks \( y \) in \( \mathcal{F}' \). Thus, \( z \) attacks \( y \) in \( \mathcal{F} \) and \( z \) defends \( x \). We conclude that \( E \) is admissible in \( \mathcal{F} \).

Let us prove that \( E \) is a complete extension of \( \mathcal{F} \). By means of contradiction, suppose that \( E \) defends \( x \) in \( \mathcal{F} \) with \( x \notin E \). Let \( y \) be an attacker of \( x \) in \( \mathcal{F}' \) such that no argument defends \( x \) against the attack of \( y \) in \( \mathcal{F}' \). This means that \( y \) attacks \( x \) in \( \mathcal{F} \). Let \( z \in E \) be such that \( z \) attacks \( y \) in \( \mathcal{F} \). We know that \( z \) does not attack \( y \) in \( \mathcal{F}' \). It must be that \( y \) attacks \( z \) in \( \mathcal{F}' \). Since we supposed no argument from \( E \) attacks \( y \) in \( \mathcal{F}' \), set \( E \) is not admissible in \( \mathcal{F}' \), contradiction. So, it must be that \( E \) is a complete extension of \( \mathcal{F} \).

**Stable semantics.**
Let $E$ be a stable extension of $F'$. Then $E$ is conflict-free in $F$. Let $x$ be an argument in exterior of $E$. Note that $E$ attacks $x$ in $F'$. Hence, $E$ attacks $x$ in $F$. Thus, $E$ is a stable extension of $F$. ■

As a consequence of the previous result, P3, P4, P5 and P6 are satisfied by R3 under complete and stable semantics.

Since P2 is satisfied under complete and stable semantics, this means that each extension of $(A, Att, \succeq \cup \succ')$ is an extension of $(A, Att, \succeq)$, which implies that P4 holds under complete semantics. Let us now show why R3 satisfies P4 under complete semantics. Let $E$ be the ground extension of $(A, Att, \succeq \cup \succ')$ under reduction R3. Since P2 is satisfied under complete semantics, and $E$ is a complete extension of $F'$, then $E$ is a complete extension of $F$. Thus, P4 is satisfied since the grounded extension is contained in every complete extension.

Preferred semantics does not satisfy P4 as shown by Figure 7.

P5 is not satisfied by stable and preferred semantics, a counter example is depicted in Figure 7.

P6 is satisfied under complete and stable semantics since it follows from P2. It is trivially satisfied by grounded semantics.

**Proposition 3.6** R3 satisfies P6 under preferred semantics.

**Proof.** Let $F$, resp. $F'$ be the argumentation graph obtained after applying R3 on $(A, Att, \succeq)$, resp. $(A, Att, \succeq \cup \succ')$. Let us first show that each admissible set $E$ of $F'$ is admissible in $F$. Suppose $E$ is admissible in $F'$. It is immediate to see that $E$ is conflict-free in $F$. Suppose now that $x \in E$ and that $y$ attacks $x$ in $F$. If $x$ attacks $y$ in $F$ the proof is over. Else, suppose that $x$ does not attack $y$ in $F$. This means that $y$ attacks $x$ in $F'$ and $x$ does not attack $y$ in $F'$. Since $E$ is admissible in $F'$, there exists $z \in E$ such that $z$ attacks $y$ in $F'$. Consequently, $z$ attacks $y$ in $F$. Hence, $E$ is admissible in $F$.

We conclude that the set of admissible sets of $F'$ is a subset of the set of admissible sets of $F$. Observe that this means that every extension of $F$ is a superset of at least one of the extensions of $F'$, and every extension of $F'$ has at least one superset that is an extension of $F$.

Let us now prove the proposition. By means of contradiction, suppose the contrary. From the above observation we deduce that there exists a maximal admissible set $E$ of $F$ (i.e. an extension of $F$) such that $E$ is not an extension of $F'$ and that there exist distinct $E_1$ and $E_2$ such that $E_1, E_2 \subseteq E$ and both $E_1$ and $E_2$ are extensions of $F'$.

Define $E' = E_1 \cup E_2$. Note that $E'$ is conflict-free in $F'$ since

- $E_1 \cup E_2 \subseteq E$,
- $E$ is conflict-free in $F$
- for each set $S$, we have: $S$ is conflict-free in $F$ if and only if $S$ is conflict-free in $F'$.

...
Figure 8. R4 falsifies several principles under all semantics

Note that since $E_1$ defends its arguments in $F'$ and $E_2$ defends its arguments $F'$ and since their union is conflict-free in $F'$, it must be that $E'$ is admissible in $F'$. Contradiction, since $E_1 \subset E'$ (this is true since $E_2$ is not a subset of $E_1$).

P7 is satisfied since removing an attack towards an argument can only improve its status [Amgoud and Vesic, 2020].

P8 is not satisfied, the counter example is the argumentation graph with two arguments $a$ and $b$, where $b$ attacks $a$ and there are no preferences. Adding the preference $a \succ b$ will not improve the status of $a$.

P9 and P10 are satisfied since the preference between $a$ and $b$ only influences the attacks between those two arguments.

We now study R4. P1 is satisfied since R4 preserves conflict-freeness.

P2, P3, P4 and P5 are not satisfied, as can be seen in figure 8. The same is a counter example for P6 for all the semantics except grounded.

**Proposition 3.7** R4 satisfies P7 under all the studied semantics.

**Proof.** Denote by $F$ (resp. $F'$) the graph obtained after applying R4 on $(A, Att, \succeq)$, resp. $(A, Att, \succeq \cup \{(a, b)\})$.

If $(b, a) \notin Att$, $F$ and $F'$ coincide. If $(a, b) \in Att$ and $(b, a) \in Att$, the proof follows from the fact that removing an attack on $a$ cannot worsen its status [Amgoud and Vesic, 2020]. In the rest of the proof, we study the case when $(b, a) \in Att$ and $(a, b) \notin Att$.

**Stable semantics.**

Let $a$ be skeptically accepted in $F$. By means of contradiction, suppose that $a$ is not skeptically accepted in $F'$; let $E$ be an extension of $F'$ such that $a \notin E$. Note that $E$ is conflict-free in $F$; it also attacks all the arguments in its exterior in $F$. Hence, $E$ is a stable extension of $F$. Contradiction.

Let $a$ be credulously accepted in $F$. Let $E$ be an extension of $F$ such that $a \in F$. Since $E$ attacks all arguments in its exterior in $F$, $E$ also attacks all arguments in its exterior in $F'$. Hence, $E$ is a stable extension of $F'$. Consequently, $a$ is not rejected in $F'$.

**Preferred semantics.**

Let $a$ be skeptically accepted in $F$. By means of contradiction, let $E$ be an extension of $F'$ such that $a \notin E$.

Case 1: $b \in E$. $E$ is conflict-free in $F$. It is also admissible in $F$. Thus, there exists $E'$ such that $E \subseteq E'$ and $E'$ is an extension of $F$. Since $b$ attacks $a$, $a \notin E'$. Thus, $a$ is not skeptically accepted in $F$, contradiction.
Case 2: $b \notin E$. Note that $E$ is admissible in $\mathcal{F}$. Let $E'$ be such that $E \subseteq E'$ and $E'$ is an extension of $\mathcal{F}$. Since $a$ is skeptically accepted, $a \in E'$. Note that $E'$ is admissible in $\mathcal{F}'$. Hence, $E$ is not an extension of $\mathcal{F}'$, contradiction.

Let $a$ be credulously accepted in $\mathcal{F}$. Let $E$ be an extension of $\mathcal{F}$ such that $a \in E$. Set $E$ is admissible in $\mathcal{F}'$, hence there exists $E'$ such that $E \subseteq E'$ and $E'$ is an extension of $\mathcal{F}'$. Thus, $a$ is not rejected in $\mathcal{F}'$.

**Complete semantics.**

Let $a$ be skeptically accepted in $\mathcal{F}$. By means of contradiction, suppose that $E$ is an extension of $\mathcal{F}'$ such that $a \notin E$.

Case 1: $b \in E$. Observe that $E$ is admissible in $\mathcal{F}$. Note that $E$ defends exactly same arguments in $\mathcal{F}$ and $\mathcal{F}'$. Thus, $E$ is an extension of $\mathcal{F}$. Contradiction.

Case 2: $b \notin E$. $E$ is admissible in $\mathcal{F}$. Note that $E$ defends $b$ in $\mathcal{F}$ if and only if $b$ is not attacked in $\mathcal{F}$. Case 2.1: $b$ is attacked in $\mathcal{F}$. Then $E$ contains all the arguments it defends in $\mathcal{F}$, hence $E$ is a complete extension of $\mathcal{F}$. Contradiction.

Case 2.2: $b$ is not attacked in $\mathcal{F}$. Then, there exists a complete extension $E'$ of $\mathcal{F}$ such that $E \subseteq E'$ and $b \in E'$. This means that $a \notin E'$, contradiction.

Let $a$ be credulously accepted in $\mathcal{F}$. Let $E$ be an extension of $\mathcal{F}$ such that $a \in E$. Note that $E$ is admissible in $\mathcal{F}'$. Hence, there exists a complete extension $E'$ in $\mathcal{F}'$ such that $E \subseteq E'$. This means that $a$ is not rejected in $\mathcal{F}'$.

**Grounded semantics.**

Let $f_C$ be the function such that, for every set of arguments $S$, $f_C(S)$ is the set of arguments defended by $S$. Then, the grounded extension is the least fixed point of $f_C(\emptyset)$ [Dung, 1995b]. Denote the grounded extension of $\mathcal{F}$ by $GE$ and the grounded extension of $\mathcal{F}'$ by $GE'$. Suppose that $a \in GE$ in $\mathcal{F}$.

Let $E_0$ be the set of non-attacked arguments of $\mathcal{F}$ and let $E_0'$ be the set of non-attacked arguments of $\mathcal{F}'$.

Since $b$ attacks $a$ in $\mathcal{F}$, $a \notin E_0$. Since $a \in GE$, $b \notin GE$.

For each $i \in \{0, 1, \ldots \}$ let $E_{i+1} = f_C(E_i)$ and $E'_{i+1} = f_C(E'_i)$. Let us prove by induction on $i$ that $E_i \subseteq E'_i$ for every $i$.

**Base:** $E_0 \subseteq E'_0$ since $a \notin E$ and $b \notin E$.

**Step:** Let $E_i \subseteq E'_i$. Let $x$ be an arbitrary argument defended by $E_i$. For each $y$ such that $y$ attacks $x$ in $\mathcal{F}$, there exists $z \in E_i$ such that $z$ attacks $y$ in $\mathcal{F}$. Note that $z \neq b$, since $b \notin GE$. Hence, since $z \in E'_i$, $x$ is defended by $E'_i$ in $\mathcal{F}'$. Thus, $E_i \subseteq E'_i$.

Consequently, $GE \subseteq GE'$. Thus, $a$ is accepted in $\mathcal{F}'$. ■

P8 is satisfied by all the semantics except grounded, since after applying R4, set $\{a\}$ is admissible. Hence, it is in at least one stable / preferred / complete extension. The counter-example for grounded semantics is the graph with two arguments $a$ and $b$, where the only attack is from $b$ to $a$. The grounded extension is the empty set, hence $a$ is rejected.

Like in the case of other reductions, P9 and P10 are satisfied.

Table 1 summarises how the proposed reductions satisfy these principles with respect to standard Dung’s semantics (i.e., complete, grounded, preferred and
Table 1. Comparison among the semantics and the principles. We refer to Dung’s semantics as follows: complete \((c)\), grounded \((g)\), preferred \((p)\), stable \((s)\). When a principle is never satisfied by a certain reduction for all semantics, we use the \(\times\) symbol. P1 refers to Principle 1, the same holds for the others.

4 Open Research Challenges for Preference in Abstract Argumentation

We believe that the concept of preference is important, because it is a natural way to represent and reason about the comparative strength of arguments. Given the limited attention preference has received in the literature of abstract argumentation, we believe that much more work needs to be done. For example, more principles for preference-based argumentation can be defined and for the four reductions, the principles of this chapter can be verified against other semantics of argumentation frameworks proposed in the literature.

In this section, we discuss four other open challenges we see in the formal study of preference in abstract argumentation. The first challenge concerns the semantics of preference-based argumentation frameworks, the second challenge concerns the kind of preference-based argumentation frameworks considered, the third challenge concerns the relation between the role of preference in abstract argumentation and in structured argumentation, and the fourth challenge concerns the dynamics of preference and argumentation.

4.1 Alternative semantics for preference in abstract argumentation

The first open challenge in the formal study on preference in abstract argumentation concerns the semantics of preference-based argumentation frameworks. In the reduction-based approach, the meaning of preference is local, in the sense that a preference between arguments \(a\) and \(b\) only affects the attack among arguments \(a\) and \(b\). Due to this locality, and the small number of reductions, the number of semantics is just a multiple of the number of semantics for Dung
argumentation frameworks.

However, if we consider a more global use of the preferences among arguments, the number of possible semantics for preference-based argumentation frameworks is much larger. For example, consider the use of preference in choice theory, or in non-monotonic logic, or in belief revision. In all these areas, the introduction of preference leads to an explosion of the number of possibilities. Likewise, the use of preference in structured argumentation increases the number and breadth of the formal approaches and results.

In the literature of formal argumentation, we are aware of only one approach by Amgoud and Vesic [2014] introducing the idea of “refining argumentation frameworks by preferences,” by which they mean that they “refine the result of the PAF,” or more precisely “they allow to choose some extensions among the set of extensions of the repaired framework.”

Kaci et al. [2018] call it the selection approach, and they illustrate it by the following example. Assume a set of extensions, and a preference relation over arguments. To get the best extensions, we need to lift the preference relation over arguments to a preference relation over sets of arguments, or extensions. It is well known that there are various ways to make this lifting more precise, see for example the work of Amgoud and Vesic [2014]. Kaci et al. [2018] use the following lifting: if argument $a$ is preferred to argument $b$, then all extensions containing argument $a$ but not $b$ are either better than all extensions containing $b$ but not $a$, or the two extensions are incomparable.

**Definition 4.1** Let $E(A, \text{Def})$ be a set of extensions according to a Dung semantics, and let $\succ$ be an order (irreflexive and transitive) over $A$, called preference relation. $E \subseteq A$ is at least as good as $E' \subseteq A$ if $\forall a, b \in A: \text{we do not have that } a \succ b \text{ and } a \in E' \setminus E \text{ and } b \in E \setminus E'$. $E$ is better than $E'$ iff $E$ is at least as good as $E'$ and $E'$ not at least as good as $E$. $E$ is best if there is no $E'$ that is better than $E$.

Kaci et al. [2018] define a semantics of preference-based argumentation by a two-step process. First, they select the extensions of the framework according to a regular abstract semantics. Then they use the preference relation to select the best extensions among them. We identify Def=Att in Dung’s AF and define $E(A, \text{Att}, \succ)$ as a selection of the best extensions of $(A, \text{Att})$.

**Definition 4.2** The extensions $E(A, \text{Att}, \succ)$ are the best extensions among $E(A, \text{Att})$ based on $\succ$.

In preference elicitation, the preferences are extracted from a user step by step [Viappiani, 2014]. Assume now that for each step where a user is queried for preference, we consider the arguments the user accepts, given the knowledge about the preference relation thus far. In such a setting, it would be quite useful if the set of accepted arguments is increasing monotonically. Moreover, the other way around, we can even consider scenarios where the interest in the extension guides the order in which the preferences are elicited.
Example 4.3 Consider the framework below under preferred or stable semantics. The left PAF prefers extension \( \{a, c\} \) over extension \( \{b, d\} \), but if we add \( b \succ a \) then both \( \{a, c\} \) and \( \{b, d\} \) are best extensions.

![Diagram of the framework](image)

This is just one of the many possibilities, and its simplicity has some drawbacks. For example, if the empty set is an extension, e.g., in complete semantics, then it is always a best extension. Just like in the case of the reductions, this illustrates that more alternatives can be defined, and a principle-based approach is needed to choose among the alternatives.

Moreover, Amgoud and Vesic [2014] consider also the combination of reduction and selection. In particular, they introduce so-called rich preference-based argumentation frameworks that allow to take into account the two roles of preferences (reduction and selection) through a two-step process. First, the preferences are used to define a reduction and calculate a set of “preliminary” extensions. Second, the preference is once again used to select the best among those preliminary extensions. The interested user is referred to the corresponding journal publication [Amgoud and Vesic, 2014] for a further discussion.

However, despite the lack of approaches in the literature, it is straightforward to generalize semantic approaches for argumentation frameworks to preference-based argumentation. For example, a common idea in abstract argumentation semantics is to use an SCC recursive algorithm [Baroni et al., 2005; Baroni et al., 2014], where SCC stands for the standard graph-theoretic notion of strongly connected component, defined as follows.

Definition 4.4 An Att-path is a sequence \( \langle a_0, \ldots, a_n \rangle \) of arguments where \((a_i, a_{i+1}) \in \text{Att}\) for \(0 \leq i < n\) and where \(a_j \neq a_k\) for \(0 \leq j < k \leq n\) with either \(j \neq 0\) or \(k \neq n\).

Let \( F = \langle A, \text{Att} \rangle \) be an AF, and let \( a, b \in A \). We define \( a \sim b \) iff either \( a = b \) or there is an Att-path from \( a \) to \( b \) and there is an Att-path from \( b \) to \( a \). The equivalence classes under the equivalence relation \( \sim \) are called strongly connected components (SCCs) of \( F \). We denote the set of SCCs of \( F \) by \( \text{SCCs}(F) \). Given \( S \subseteq A \), we define \( D_F(S) := \{ b \in A | \exists a \in S : (a, b) \in \text{Att} \land a \not\succ b \} \).

Cramer and van der Torre [2019] define the SCC-recursive scheme as a function that maps a semantics \( \sigma \) to another semantics \( \text{scs}(\sigma) \), such that \( \sigma \) is applied only to the strongly connected components. The simplest case is defined as follows.
Definition 4.5 Let $\sigma$ be an argumentation semantics. The argumentation semantics $\text{scc}(\sigma)$ is defined as follows. Let $F = \langle A, \text{Att} \rangle$ be an AF, and let $S \subseteq A$. Then $S$ is an $\text{scc}(\sigma)$-extension of $F$ iff either

- $|\text{SCCs}(F)| = 1$ and $S$ is a $\sigma$-extension of $F$, or
- $|\text{SCCs}(F)| > 1$ and for each $C \in \text{SCCs}(F)$, $S \cap C$ is an $\text{scc}(\sigma)$-extension of $F|_{C \setminus \text{DF}(S)}$.

We refer the interested reader to the extensive literature on SCC recursion [Baroni et al., 2005; Baroni et al., 2014]. What we would like to emphasize here is how this idea can be used as well for preference-based argumentation. For example, we can use the preferences only when we consider the strongly connected components, not when we consider the recursion.

4.2 Preference-based argumentation frameworks

The second open challenge in the formal study on preference in abstract argumentation concerns the representation of preference-based argumentation frameworks. On the one hand, the preference relation can be represented by a numerical function rather than comparative preferences. As this is studied in an accompanying paper in this handbook, we do not consider this possibility in this chapter. On the other hand, we can impose various kinds of constraints on the attack and preference relation in preference-based argumentation frameworks.

For preference, this is well known. Much has been written in the formal literature on preference and preference logic about intransitive preference, totally ordered preference, and so on.

Less is known about constraints on the attack relation. In Dung’s argumentation, it is well known that a symmetric attack relation leads to a collapse of many semantics, and in general to a trivialization of the theory. For this reason, symmetric attacks are rarely studied. However, a symmetric attack also has advantages. One of the questions in abstract argumentation is to distinguish the different interpretations of directional attack. A symmetric attack, however, is very similar to negation in logic and thus more easily understood and manipulated.

For example, consider an argumentation theory in which each argument is represented by a propositional formula, and ‘argument A attacks argument B’ is defined as ‘$A \land B$ is inconsistent’. Unfortunately, such a simple argumentation theory is not very useful, since the attack relation is symmetric, and the various semantics reduce to, roughly, one of the following two statements: ‘an argument is acceptable iff it is part of all/some maximal consistent subsets of the set of arguments.’ However, such a simple argumentation theory is useful again when we add the additional condition that argument A is at least as preferred as argument B. It may also be worthwhile to consider the generalizations of Dung’s framework in propositional argumentation [Boella et al., 2005].

As the notion of symmetric attack is considered to be very important, Kaci et al. [2006] introduce a new term for it, and refer to it as the conflict relation.
The new preference-based argumentation framework considers a conflict and a preference relation. The conflict relation should not be interpreted as an attack relation, since a conflict relation is symmetric, and an attack relation is usually asymmetric.

**Definition 4.6 (Conflict+preference argumentation framework)** A conflict+preference argumentation framework is a triplet \( \langle A, C, \succeq \rangle \) where \( A \) is a set of arguments, \( C \) is a binary symmetric conflict relation defined on \( A \times A \) and \( \succeq \) is a (total or partial) pre-order (preference relation) defined on \( A \times A \).

An acyclic argumentation framework is an argumentation framework in which the attack relation is acyclic, a symmetric argumentation framework is an argumentation framework in which the attack relation is symmetric, etc. Kaci et al. [2006] define an acyclic strict attack relation as follows. Assume the attack relation is such that there is an attack path where argument \( A_1 \) attacks argument \( A_2 \), argument \( A_2 \) attacks argument \( A_3 \), etc, and argument \( A_n \) attacks argument \( A_1 \), then we have that all the arguments in the attack path attack the previous one. Consequently, if argument \( A \) strictly attacks \( B \) if \( A \) attacks \( B \) and not vice versa, then the strict attack relation is acyclic.

**Definition 4.7 (Acyclic argumentation framework)** A strictly acyclic argumentation framework is an argumentation framework \( \langle A, R \rangle \) in which the attack relation \( R \subseteq A \times A \) satisfies the following property:

If there is a set of attacks \( A_1 R A_2, A_2 R A_3, \cdots, A_n R A_1 \) then we have that \( A_2 R A_1, A_3 R A_2, \cdots, A_1 R A_n \).

Kaci et al. [2006] show that strictly acyclic argumentation frameworks are characterized by conflict+preference argumentation frameworks. For this they use an unusual kind of reduction, defined as follows.

**Definition 4.8** Let \( \langle A, R \rangle \) be an argumentation framework and \( \langle A, C, \succeq \rangle \) a conflict+preference argumentation framework. We say that \( \langle A, C, \succeq \rangle \) represents \( \langle A, R \rangle \) if for all arguments \( A \) and \( B \) of \( A \), we have \( A R B \) if \( A C B \) and \( A \succeq B \). We also say that \( R \) is represented by \( C \) and \( \succeq \).

**Theorem 4.9** \( \langle A, R \rangle \) is a strictly acyclic argumentation framework if and only if there is a conflict+preference argumentation framework \( \langle A, C, \succeq \rangle \) that represents it.

The preference-based argumentation theory introduced by Kaci and colleagues is a variant of Reduction 1, their representation theorem does not hold if Reduction 1 is used. As far as we know it is an open problem which properties the attack relation satisfies (if the defeat relation is symmetric). More generally, the question may be raised which principles hold only for preference-based argumentation frameworks with a symmetric attack relation, but not in general.
4.3 Preference in argumentation

The third challenge concerning preference in abstract argumentation is to relate it to the different ways proposed in the literature to compute a preference relation over arguments depending on the internal structure of the arguments. We are aware only of one formal result, which we present in the first subsection. The other subsections present other ways in the literature in which preference is calculated in structured argumentation, and for which no corresponding result in abstract argumentation has been provided yet.

4.3.1 Structured arguments & Implicit priorities

The introduction of preferences in argumentation theory was first proposed by Simari and Loui [1992]. In their framework, an argument is structured in the form of a tuple \( \langle H, h \rangle \), where \( H \) is a set of formulas and \( h \) is a formula. \( H \) is called the support of the argument and \( h \) is its conclusion. By abuse of language we say that \( \langle H, h \rangle \) is an argument for \( h \). Inspired by non-monotonic reasoning, the authors build the arguments from a set of defeasible knowledge, denoted by \( D \). They also consider a consistent set of knowledge denoted by \( K \). Generally \( K \cup D \) is inconsistent. A pair \( \langle H, h \rangle \) is an argument if and only if it satisfies the following properties:

1. \( H \subseteq D \) and \( h \) is a formula of the language,
2. \( K \cup H \vdash h \),
3. \( K \cup H \) is a consistent subset of \( K \cup D \),
4. \( \exists \ H' \subset H \) such that \( K \cup H' \vdash h \).

We say that an argument can be activated if the premise of each rule in its support is true. This notion defines the specificity between two arguments. An argument is more specific than another argument if and only if each time the former can be activated the latter can also be activated, but the reverse is not true. The specificity relation defines the preference relation. Being more specific makes an argument preferred.

4.3.2 Structured arguments & Explicit priorities

Priorities in the above setting are implicitly represented by the specificity principle. Amgoud et al. [1996] consider explicit priorities. More precisely, in their approach, knowledge is represented by a set of weighted propositional logic formulas \( K \). Let \( K = \{ (\phi_i, \alpha_i) | i = 1, \ldots, n \} \), where \( \phi_i \) is a propositional logic formula and \( \alpha_i \in (0, 1] \) is the certainty/priority degree associated with \( \phi_i \). An argument is also a pair \( \langle H, h \rangle \). Conditions (1)-(4) given above apply in this case too, where \( K = \emptyset \) and \( D \) is replaced with \( K^* = \{ \phi_i | (\phi_i, \alpha_i) \in K \} \).

One can then construct a function \( w : A \rightarrow [0, 1] \), where \( w(\langle H, h \rangle) \) depends on the weight of formulas involved in \( H \) [Amgoud et al., 1996]. Thus, an argument is preferred iff it is preferred w.r.t. \( w \).
4.3.3 Abstract arguments and values

In some applications, the arguments need to be compared not on the basis of their internal structure but with respect to the viewpoints or decisions they promote. This may be due to the fact that the internal structure of the arguments is not available or because the values must be considered. This is particularly true in persuasion dialogues when the preference over values induces the preference over arguments promoting those values [Bench-Capon, 2003]. The more important the value, the more preferred the argument is. Thus, if two arguments are conflicting then the argument promoting a preferred value is accepted. For example, suppose that two parents discuss whether their son can watch the soccer game on the TV or whether he should prepare for his exam. Watching the game allows their son to discuss it with his friends, which promotes his sociability. On the other hand, preparing for his exam promotes his education. If the parents consider that sociability is not more important than education, then the child should prepare for his exam.

Bench-Capon [2003] developed an argumentation framework which models the above considerations (see Chapter 4 of this Handbook for a detailed discussion). Like Dung’s framework he considers abstract arguments. Moreover he considers (i) a set of values promoted by the arguments and (ii) a set of audiences, following Perelman [1980], where each audience corresponds to a preference relation over values.

**Definition 4.10 (Value-based argumentation framework) [Bench-Capon, 2003]**

A Value-based Argumentation Framework (VAF) is a five-tuple, \( \langle A, \text{Att}, V, \text{val}, D \rangle \), where \( A \) is a set of arguments, \( \text{Att} \) is an attack relation, \( \text{val} \) is a function which maps from elements of \( A \) to elements of \( V \) and \( D \) is the set of possible audiences. An audience specific argumentation framework is a five-tuple, \( \langle A, \text{Att}, V, \text{val}, >_{ad} \rangle \), where \( ad \in D \) is an audience and \( >_{ad} \) is a partial order over \( V \).

It is worth noticing that \( \text{Att} \) is independent of \( \text{val} \), in the sense that two arguments promoting the same value may be related with the attack relation.

**Definition 4.11 (Audience-specific value-based argumentation framework) [Bench-Capon, 2003]**

\( \langle A, \text{Att}, V, \text{val}, >_{ad} \rangle \) represents \( \langle A, \text{Att}, > \rangle \) if and only if \( \forall A, B \in A, \text{ we have} \)

\[ a > b \text{ if and only if } \text{val}(a) >_{ad} \text{val}(b). \]

Concerning the existence of audience-specific value-based argumentation frameworks representing a preference-based argumentation framework, the situation is the same as between preference-based argumentation frameworks and argumentation frameworks. Each audience-specific value-based argumentation framework represents precisely one preference-based argumentation framework,
and each preference-based argumentation framework is represented by an equivalence class of alphabetic variants of audience-specific value-based argumentation frameworks [Kaci and van der Torre, 2008]. The acceptable extensions of an audience-specific value-based argumentation framework are again simply the acceptable extensions of the unique preference-based argumentation framework it represents.

4.3.4 Abstract arguments & NMR on preferences over values
In Bench-Capon’s framework an argument promotes at most one value. However, in practice it may be the case that arguments promote multiple values. Moreover, in Bench-Capon’s framework, a value $v_1$ being more important than (or preferred to) a value $v_2$ is interpreted as any argument promoting $v_1$ being preferred to any argument promoting $v_2$. One can also imagine other ways to compare the arguments promoting $v_1$ and $v_2$. Kaci and van der Torre [2008] extend Bench-Capon’s framework in order to take into account the previous considerations. More specifically, they consider (i) arguments promoting multiple values, and (ii) various kinds of preferences over values.

4.3.5 Valued Preference-based Argumentation Frameworks with Varied Strength Defeats
Kaci & Labreuche [2014] developed a preference-based argumentation framework in which the preference relation over arguments has varied strengths. The valued preference relation can be computed from a Boolean preference relation or from a valuation of arguments. Associated with an attack relation, the valued preference relation leads to a defeat relation with varied strengths.

4.3.6 Argument-based preferences over arguments
Modgil [2009] developed a preference-based argumentation framework, called extended argumentation framework, in which the arguments are abstract entities and the preference relation over the set of arguments is not defined by external information (e.g., ordered values or information pervaded with implicit or explicit priorities). Preferences over arguments are supported by arguments. More precisely, Modgil uses Reduction 1 in which the condition $\neg(b \succ a)$ is supported by an argument. In other words, $(a, b) \in \text{Def}$ if and only if $(a, b) \in \text{Att}$ and there is no argument claiming that $b$ is preferred to $a$.

**Definition 4.12 (Extended argumentation framework [Modgil, 2009])**
An extended argumentation framework is a three-tuple $(A, \text{Att}, \mathcal{H})$ such that $A$ is a finite set of arguments and

- $\text{Att} \subseteq A \times A$ is an attack relation,
- $\mathcal{H} \subseteq A \times \text{Att}$: $(a, (b, d))$ stands for “$a$ claims that $d$ is preferred to $b$”,
- if $(a, (b, d)), (a', (d, b)) \in \mathcal{H}$ then $(a, a'), (a', a) \in \text{Att}$. 
4.4 Dynamics of preference and argumentation

The fourth open challenge in preference-based abstract argumentation is the dynamics of argumentation. In the literature, often two kinds of dynamics are distinguished. One kind of dynamics can be found in dialogue, where the speech acts of the agents may affect the knowledge bases of the agents. The other kind of dynamics can be found in some principles, when they compare distinct argumentation frameworks.

Some of the principles discussed in this chapter are dynamic in a special way: the attack relation is considered to be fixed, but only the preference relation changes. Whereas the dynamics of argumentation frameworks may be criticized, we believe that the dynamics of the preference relation occurs naturally in many applications e.g. recommender systems [Jannach et al., 2010].

5 Related research

Amgoud and Vesic [2014] have proposed a new PAF which satisfies two requirements: conflict-freeness of extensions with respect to the attack relation (this is Principle 1 in our analysis), and, in the absence of critical attacks, the extensions of a preference-based argumentation framework coincide with the extensions of Dung’s argumentation framework. Moreover the PAF handles critical attacks following Reduction 2. The second requirement can be roughly reformulated as: if the preference relation is empty, extensions of a PAF and Dung’s framework coincide.

To compute extensions, Amgoud and Vesic [2014] define a preference relation over the powerset of arguments using the so-called democratic and elitist relations. Maximal conflict-free subsets are extensions of the PAF (grounded, preferred and stable extensions). The Definition 4.1 of Kaci et al. [Kaci et al., 2018] is incomparable with democratic and elitist relations. Definition 4.1 applies on the PAF with ignoring the associated preference relation while [Amgoud and Vesic, 2014] applies democratic and elitist relations on the PAF with Reduction 2.

There is a striking similarity at the abstract level between preference based argumentation and support in bipolar argumentation, as both can be seen as reductions [Yu and Torre, 2020].

Other approaches have been proposed in the literature to reason over preferences in argumentation. Among them, in [Prakken and Sartor, 1997], the underlying language is Extended Logic Programming (ELP), which includes strict rules and defeasible rules, while the preference information is provided under the form of an ordering on the defeasible rules. They study two different cases with respect to the nature of this ordering. In the first one, the ordering is fixed and indisputable (i.e., strict priorities), while in the second case priorities are themselves defeasibly derived as conclusions within the system. To support defeasible priorities, they allow stating rules and constructing arguments about priorities.

The argumentation framework proposed in [Governatori and Maher, 2000]
uses the language of Defeasible Logic. In this framework, the rule priority relation of Defeasible Logic is used to determine whether an argument is defeated by a counter-argument.

The framework proposed in [Kakas and Moraitis, 2003; Kakas and Moraitis, 2006] also includes the notion of dynamic priorities in the context of negotiating agents. Roles and context define in a natural way dynamic preferences on the decisions implied by the negotiation strategies of the agents as their environment changes. The underlying monotonic logic includes a special type of rules that are used to give priority between competing rules in case of conflict. Based on these rules, they build arguments on priorities, and reason with them to give preference to specific arguments in the system.

Similarly, the framework proposed in [Bikakis and Antoniou, 2010] also adopts Defeasible Logic as the underlying formalism for building arguments. However, it assumes a distributed argumentation system, in which each agent has its own knowledge base and agents create arguments by combining their local knowledge with the beliefs of other agents. The preferences among arguments in this case are derived from the preferences of an agent on the agents it imports information from.

In [Cyras and Toni, 2016], the authors introduce a new approach to handling preferences in Assumption-Based Argumentation (ABA) called ABA+. More precisely, ABA+ assumes preferences on the object level (i.e., over assumptions) and incorporates them directly into the definition of attack, rather than assuming preferences on the meta level (e.g., over arguments). ABA+ also allows for preferences in generic ABA frameworks, as opposed to allowing for preferences only in flat ABA frameworks.

[Brewka et al., 2013] introduce static and dynamic preferences into Abstract Dialectical Frameworks (ADFs). Roughly, they handle dynamic preferences as follows: they first guess a (stable, preferred, grounded) extension $M$. Some nodes in $M$ carry the preference information. They extract this information and check whether $M$ can be reconstructed under the preference information, thus verifying that the preferences represented in the model itself were taken into account adequately.

There are also works that deal with preferences in the context of ranking-based / gradual argumentation semantics [Amgoud et al., 2017], but we do not study them in this chapter.

6 Summary

In this paper, we presented a principle-based analysis of the semantics of preference-based argumentation frameworks. We considered four reductions to move from PAFs to a Dung-like abstract argumentation on which standard semantics can be applied to compute the set of accepted arguments, and we proposed a set of ten principles we used to study the considered reductions.

The results of this paper give rise to many new research questions. Many more principles can be defined in our framework (e.g., following [Rienstra et al.,
2015), and used in the analysis. In particular, it is striking that many principles have a dynamic flavor, and we conjecture that many approaches to dynamics of argumentation [Booth et al., 2013] can be used as a source for principles. We are in particular interested in principles that distinguish the various PAF semantics. We believe that there are not many new reductions to be found, but more PAF semantics can be defined not based on reductions. We observe that the resolution-based family of abstract argumentation semantics [Baroni et al., 2011] seems also related to Reduction 3 introduced in this paper.

BIBLIOGRAPHY


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