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Guillaume Parent, Stephane Duchesne, Patrick Dular. Determination of Flux Tube Portions by Adjunction of Electric or Magnetic Multivalued Equipotential Lines. *IEEE Transactions on Magnetics*, 2017, 53 (6), pp.7401704. 10.1109/TMAG.2017.2661401 . hal-03350742

**HAL Id: hal-03350742**

**<https://univ-artois.hal.science/hal-03350742>**

Submitted on 16 May 2022

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# Determination of Flux Tube Portions by Adjunction of Electric or Magnetic Multivalued Equipotential Lines

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In 2-D electrostatic and magnetostatic finite element problems, portions of flux tubes are defined by field lines together with their orthogonal lines. These two sets of lines can be obtained as the isovalue lines of scalar and vector potentials. One set is related to a multivalued potential that, when made single-valued via the definition of cuts, undergoes discontinuities, resulting in lines interruptions at the cuts. A method is proposed for obtaining continuous isovalue lines for the multivalued potentials. This then helps in locating selected portions of flux tubes and quantifying their equivalent properties, i.e., capacitances or reluctances, to be used as is or in equivalent circuits.

*Index Terms*—Finite element analysis, field lines, flux tubes

## I. INTRODUCTION

WHEN modeling an electromagnetic device, the choice of one method or the other depends on several parameters such as the level of desired accuracy, computational time or complexity of the physical phenomena. The equivalent circuit modeling approach, whether electric or magnetic, usually offers the best balance between computation time and accuracy of the results, which is why it is intensively used for fast modeling [1], failure analyzes [2] and design-optimization [3]. Nevertheless, both accuracy and ruggedness of these studies greatly depend on the determination of the lumped parameters constituting the equivalent circuit. They can be determined by analytical relations, tubes and slices method [4] or from Finite Element (FE) modeling [5].

The method presented in this paper uses the latter approach but, differently from [5], it uses both static dual 2-D formulations to accurately determine flux tube portions, delimited by both flux walls and gates, that are related to lumped parameters. Indeed, an electromagnetic problem is usually solved through either a scalar or a vector potential formulation. In 2-D and in case of a total potential, the associated field is either parallel or perpendicular to its isovalue lines, which makes the determination of such lines a key point for the characterization of flux tubes portions. For a given problem, obtaining both sets of lines requires the use of dual formulations. Whereas the commonly used formulations (with electric scalar potential in electrostatics and magnetic vector potential in magnetostatics) involve single-valued potentials, their dual formulations (with electric vector potential and magnetic scalar potential, respectively) involve multivalued potentials, i.e., presenting discontinuities through some cut lines making their support domain simply connected [6]. The aim of this work is the development of a method allowing to adjunct the parts of the isovalue lines of the multivalued potentials at the cuts in order to be able to exploit them as boundaries of flux tubes portions.

The complementary portions are directly obtained from the complementary classical single-valued potentials.

Electrostatic and magnetostatic test problems are solved to illustrate and validate the developed methodology.

## II. STATIC DUAL FORMULATIONS AND FLUX TUBES

### A. Electrostatic and magnetostatic problems

Electrostatic and magnetostatic problems in a bounded domain  $\Omega \subset \mathbb{R}^2$  of boundary  $\Gamma$  are governed by :

$$\text{curl } \mathbf{e} = 0, \text{div } \mathbf{d} = \rho, \mathbf{d} = \epsilon \mathbf{e} \quad (1a)$$

$$\text{curl } \mathbf{h} = \mathbf{j}_s, \text{div } \mathbf{b} = 0, \mathbf{b} = \mu \mathbf{h} \quad (1b)$$

where  $\mathbf{e}$  is the electric field,  $\mathbf{d}$  is the electric flux density,  $\mathbf{h}$  is the magnetic field,  $\mathbf{b}$  is the magnetic flux density,  $\mathbf{j}_s$  is a source current density,  $\rho$  is a source volume charge density,  $\epsilon$  is the electric permittivity and  $\mu$  is the magnetic permeability. Their weak formulations are obtained from Green's identities (2):

$$(\mathbf{f}, \text{grad } g)_\Omega + (\text{div } \mathbf{f}, g)_\Omega = \langle \mathbf{n} \cdot \mathbf{f}, g \rangle_\Gamma \quad (2a)$$

$$(\text{curl } \mathbf{f}, \mathbf{g})_\Omega + (\mathbf{f}, \text{curl } \mathbf{g})_\Omega = \langle \mathbf{n} \times \mathbf{f}, \mathbf{g} \rangle_\Gamma \quad (2b)$$

where  $\mathbf{f}, \mathbf{g} \in \mathbf{H}^1(\Omega)$ ,  $g \in H^1(\Omega)$ ,  $(\cdot, \cdot)_\Omega$  and  $\langle \cdot, \cdot \rangle_\Gamma$  denote a volume integral in  $\Omega$  and a surface integral on  $\Gamma$  of the product of their vector field arguments respectively and  $\mathbf{n}$  is the unit normal vector exterior to  $\Omega$ . The domain  $\Omega$  is composed of conducting and nonconducting parts  $\Omega_c$  and  $\Omega_c^C$  as  $\Omega = \Omega_c \cup \Omega_c^C$ . Moreover,  $\Omega_s$  denotes the source parts for  $\rho$  or  $\mathbf{j}_s$ , with  $\Omega_s \subset \Omega_c$  in electrostatics and  $\Omega_s \subset \Omega_c^C$  in magnetostatics.

From each of the systems (1a) or (1b), two dual formulations can be established – one involving a single-valued potential, the other one involving a multivalued potential – depending on the selected strong and weak formulations, and thus whether (2a) or (2b) is used [7,8].

### B. Single-valued potentials formulations

The single-valued potentials formulations, either in electrostatics or in magnetostatics, are the most commonly used, essentially for their ease of implementation and use.

In electrostatics, the  $e$ -conform formulation is obtained by introducing the electric scalar potential  $v$  such that  $\mathbf{e} = -\text{grad } v$ . Then, in  $\Omega_c^C$ , i.e., where  $\rho = 0$ , the weak form of  $\text{div } \mathbf{d} = 0$  can be obtained from (2a):

$$(-\epsilon \text{grad } v, \text{grad } v')_{\Omega_c^C} - \langle \mathbf{n} \cdot \mathbf{d}, v' \rangle_{\Gamma_d} = 0, \forall v' \in F_e^0(\Omega_c^C) \quad (3)$$

where  $F_e^0(\Omega_c^C)$  is the space of admissible potential  $v$  and  $v'$  is its associated test function [7].

In magnetostatics, the  $b$ -conform formulation is obtained by introducing the magnetic vector potential  $\mathbf{a}$  such that  $\mathbf{b} = \text{curl } \mathbf{a}$ , with a gauge condition on  $\mathbf{a}$  to ensure its uniqueness. The weak form of  $\text{curl } \mathbf{h} = \mathbf{j}_s$  can be obtained from (2b):

$$(\mu^{-1} \text{curl } \mathbf{a}, \text{curl } \mathbf{a}')_{\Omega} - (\mathbf{j}_s, \mathbf{a}')_{\Omega_s} + \langle \mathbf{n} \times \mathbf{h}, \mathbf{a}' \rangle_{\Gamma_h} = 0 \quad \forall \mathbf{a}' \in F_b^1(\Omega) \quad (4)$$

where  $F_b^1(\Omega)$  is the space of admissible potential  $\mathbf{a}$  and  $\mathbf{a}'$  is its associated test function [8].

### C. Multivalued potentials formulations

The multivalued potentials formulations, in duality with the formulations presented in section II-B, usually require the introduction of source fields related to their associated sources [6,7,9]. These are non-zero in the source regions  $\Omega_s$  and can be extended to be non-zero in  $\Omega_s^C = \Omega \setminus \Omega_s$ , allowing the initially multivalued total potentials to become single-valued reaction potentials.

In electrostatics, the source field  $\mathbf{d}_s$  allows ensuring  $\text{div } \mathbf{d}_s = \rho$  in  $\Omega_c$  and  $\text{div } \mathbf{d}_s = 0$  in  $\Omega_c^C$ . Electric flux density  $\mathbf{d}$  can then be expressed as the association of such a source field and a divergence-free reaction field, expressed as the curl of an electric vector potential  $\mathbf{u}$  (with a gauge condition), i.e.,  $\mathbf{d} = \mathbf{d}_s + \text{curl } \mathbf{u}$  [7], leading to the  $d$ -conform formulation:

$$(\epsilon^{-1} \mathbf{d}_s, \text{curl } \mathbf{u}')_{\Omega_c^C} + (\epsilon^{-1} \text{curl } \mathbf{u}, \text{curl } \mathbf{u}')_{\Omega_c^C} + \langle \mathbf{n} \times \mathbf{e}, \mathbf{u}' \rangle_{\Gamma_e} = 0, \forall \mathbf{u}' \in F_d^1(\Omega_c^C) \quad (5)$$

where  $F_d^1(\Omega_c^C)$  is the space of admissible potential  $\mathbf{u}$  and  $\mathbf{u}'$  is its associated test function. The same procedure is followed in magnetostatics [8]. In presence of a nonzero current source  $\mathbf{j}_s$  in  $\Omega_s$ , a source field  $\mathbf{h}_s$  is defined such that  $\text{curl } \mathbf{h}_s = \mathbf{j}_s$ . The reaction field can thus be expressed as the gradient of a magnetic scalar potential  $\phi$ , leading to  $\mathbf{h} = \mathbf{h}_s - \text{grad } \phi$ . The resulting  $h$ -conform formulation is:

$$(\mu \text{grad } \phi, \text{grad } \phi')_{\Omega} - (\mu \mathbf{h}_s, \text{grad } \phi')_{\Omega} + \langle \mathbf{n} \cdot \mathbf{b}, \phi' \rangle_{\Gamma_b} = 0 \quad \forall \phi' \in F_h^0(\Omega) \quad (6)$$

where  $F_h^0(\Omega)$  is the space of admissible potential  $\phi$  and  $\phi'$  is its associated test function.

### D. Using potentials iso-value lines as flux tubes boundaries

For simplicity of presentation, one denotes  $\mathbf{F}$  either the electric or magnetic flux density,  $\mathbf{f}$  either the electric or magnetic vector potential, i.e.,  $\mathbf{u}$  or  $\mathbf{a}$ ,  $\mathbf{G}$  either the electric or magnetic field, and  $g$  either the electric or magnetic scalar potential, i.e.,  $v$  or  $\phi$ . In 2-D, with electric and magnetic fields

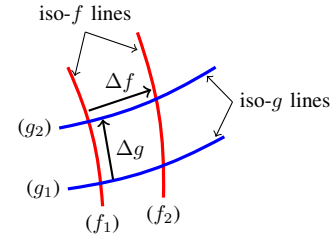


Fig. 1. Flux tube boundaries definition

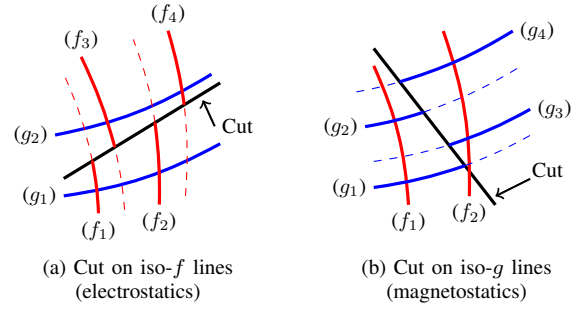


Fig. 2. Discontinuities of  $f$  and  $g$  iso-value lines on cuts

in the  $xy$ -plane,  $\mathbf{f}$  can be written  $\mathbf{f} = f_z \mathbf{z} = f \mathbf{z}$  where  $\mathbf{z}$  is the unit vector perpendicular to the studied plane. This implies that gauge condition  $\text{div } \mathbf{f} = 0$  is implicitly satisfied. In regions where potentials  $f$  and  $g$  are total (with a zero source field), one can write:

$$\mathbf{F} = \text{curl } \mathbf{f} = \text{grad } f \times \mathbf{z} \quad (7a)$$

$$\mathbf{G} = -\text{grad } g \quad (7b)$$

From (7a), it is straightforward to deduce that iso- $f$  lines and  $\mathbf{F}$  are colinear, i.e., field lines of  $\mathbf{F}$  are iso- $f$  lines. In other words, iso- $f$  lines represent flux walls for  $\mathbf{F}$ . Similarly, it is straightforward to deduce from (7b) that iso- $g$  lines are normal to  $\mathbf{G}$ , i.e., iso- $g$  lines represent flux gates for  $\mathbf{G}$ . Hence, in the case of continuous iso-lines, the area bounded by iso- $f$  and iso- $g$  lines defines a flux tube (Fig. 1) and can then be related to its equivalent property, i.e., a capacitance or a reluctance in case of an electrostatic or a magnetostatic problem, respectively. Because (7a) only makes sense in 2-D, the method is not applicable as such to 3-D cases.

Among the defined potentials, those that can be multivalued are  $f \equiv \mathbf{u} = u \cdot \mathbf{z}$  in electrostatics and  $g \equiv \phi$  in magnetostatics. In general, multivalued potentials are not total in  $\Omega_c^C$  because, as mentioned in section II-C,  $d$ -conform and  $h$ -conform formulations imply the introduction of source fields, that are non-zero in  $\Omega_s^C$ . These can however be substituted with discontinuities of the associated potential  $f \equiv u$  or  $g \equiv \phi$  through cuts making  $\Omega_s^C$  simply connected [6,9], which thus makes that potential total in the whole  $\Omega_c^C$ . In electrostatics, the discontinuity  $\Delta u_{cut_c}$  through a cut  $c$  is the total electric charge carried by the source conducting region connected only to that cut, with the plus or minus sign depending on the cut orientation. If more than one cut is connected to a source region, each of the related cut discontinuities is a portion of the carried total charge, their sum being the total charge. Similarly,

in magnetostatics, the discontinuity  $\Delta\phi_{cut_c}$  through a cut  $c$  is the signed total electric current carried by the conducting region connected only to that cut or a portion of that current in case of multiple cuts.

The discontinuities of the potentials do not allow the direct definition of flux tubes in the vicinity of the cuts regions (Fig. 2). The adjunction of associated portions of isovalue lines on both sides of the cuts has to be done, which asks for a particular treatment as developed hereafter.

### III. ADJUNCTION OF MULTIVALUED POTENTIAL LINES

From a calculated FE-wise distribution of a multivalued potential, e.g.,  $f \equiv u$ , in  $\Omega_c^C$ , pairs of additional FE-wise distributions are created by shifting the initial potential to plus or minus cut discontinuity  $\Delta f_{cut_c}$ , for each cut  $c$ . The initial distribution with all the shifted distributions for all cuts fill a database of distributions  $D = \{f, \bigcup_{\text{all cuts}} \{f + \Delta f_{cut_c}, f - \Delta f_{cut_c}\}\}$ .

Then, a line  $\gamma$  is selected from which iso- $f$  lines are starting for a certain set  $S = \{f_0, \dots, f_n\}$  of  $n + 1$  discrete values  $f_i, i = 0, \dots, n$ , regularly incremented with a constant  $\Delta f$ , i.e.,  $f_{i+1} = f_i + \Delta f, i = 0, \dots, n - 1$ . Point  $p_i$  denotes the point on  $\gamma$  at which value  $f_i$  is found. One has  $f_n - f_0 = \Delta f_{cut_c}$ , and thus  $\Delta f = \Delta f_{cut_c}/n$ , when the starting surface is the one of a particular source region connected only to cut  $c$ . This selected set can possibly be reduced to values of  $f$  encountered along some particular surfaces in the studied problem, e.g., the surfaces of the stator and rotor teeth of an electrical machine for  $g \equiv \phi$  in place of  $f$ , to focus on the flux tubes in a particular region, between these particular surfaces. For each value  $f_i$  in the considered set  $S$ , the successive unique FE-wise portions  $P_{i,j}$  ( $j = 0, \dots$  is the portion index) of the related iso- $f_i$  line are searched for in the database  $D$ , starting with the portion that originates from point  $p_i$  on the selected surface  $\gamma$ . Each added portion is the one in  $D$  starting from the end of the last added portion. Its end becomes the starting point for the next portion up to the end of the iso- $f_i$  line, usually located on the boundary of another source region or a symmetry plane. A summary of the whole procedure is given in Table I.

TABLE I  
POST-PROCESSING ADJUNCTION PROCEDURE

Step	Action
1)	Build distribution set $D = \{f, \bigcup_{\text{all cuts}} \{f + \Delta f_{cut_c}, f - \Delta f_{cut_c}\}\}$
2)	Select a line region $\gamma$ in $\Omega_c^C$ with its set of discrete values $S = \{f_0, \dots, f_n\}$ of $f$ , at respective positions $\{p_0, \dots, p_n\}$
3)	For each $f_i$ in $S$ index $j = 0$ , point $p_{\text{start}} = p_i$ Do Search the FE-wise portion of iso- $f_i$ line in $D$ starting at $p_{\text{start}}$ If portion found $P_{i,j} =$ found portion $j = j + 1, p_{\text{start}} =$ end point of found portion Else next $f_i$ While portion found

## IV. APPLICATION EXAMPLES

### A. Two-wire cable capacitance determination

As an electrostatic illustration example, a two-wire cable is considered. The linear capacitance of such a cable is:

$$C_{\text{analytical}} = \frac{\pi\epsilon_0\epsilon_r}{\ln\left(\frac{D}{r}\right)} \quad (8)$$

where  $D$  is the distance between the wires centers and  $r$  is their radius.

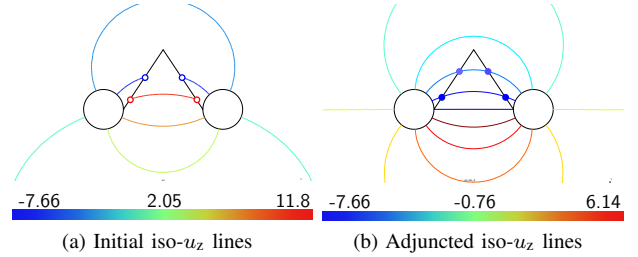


Fig. 3. Iso- $u_z$  lines for a two-conductor electrostatic problem (a cut (poly)line joins the conductors)

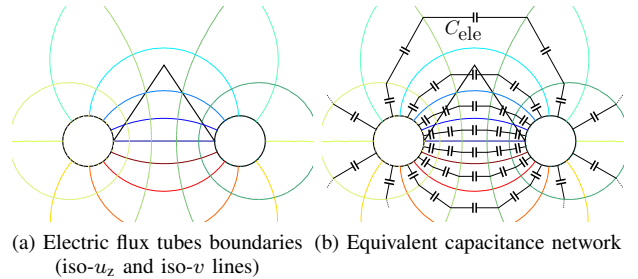


Fig. 4. Electric flux tubes boundaries and equivalent capacitance network

The result of the problem solved with the  $d$ -conform formulation is presented in Fig. 3a. As stated in section II-D, the iso- $u_z$  lines (with  $u_z = u$ ) undergo a discontinuity through the cut linking the two conductors and cannot be used as flux walls as is (Fig. 3a). The lines portions are then adjuncted following the method described in section III to obtain continuous lines (Fig. 3b). At last, the flux tubes boundaries are properly defined by solving the problem with the  $e$ -conform formulation and combining both sets of isovalue lines, i.e.,  $u_z$  and  $v$  (Fig. 4a). For each flux tube portion, an elementary capacitance  $C_{\text{ele}} = \frac{|\Delta u_z|}{|\Delta v|}$  can be defined, where  $\Delta u_z$  and  $\Delta v$  are the fixed potential differences between its two flux walls and its two flux gates, respectively (Fig. 4b). The two-wire cable capacitance can be computed by properly associating the elementary ones, i.e., in parallel along flux gates lines at first and then in series along flux walls lines. As a numerical application, the capacitance obtained for  $D = 6$  mm and  $r = 1$  mm is  $1.5525 \times 10^{-11}$  Fm $^{-1}$  and  $1.5407 \times 10^{-11}$  Fm $^{-1}$  by using (8) and the flux tube method, respectively. It is worth noting that since the adjunction procedure is performed at the post-processing step, its accuracy only depends on the accuracy of the original finite element computations. Moreover, the distribution in the

studied domain of potential values used to plot the iso-lines are obtained by classic interpolation from the basis functions. In other words, if the  $e$ -conform and  $d$ -conform computations preceding the adjunction procedure give valid results then the adjunction procedure also will, and it is totally robust no matter the geometry of the studied case.

Moreover, it can also be noted that due to its capability to discretize  $\Omega_c^C$  into elementary portions, this method allows to compute the capacitance of a specific part of the studied domain instead of the global one, such as some insulated regions or regions with fringing field. This capability is highlighted in the following section.

### B. Airgap reluctance identification

As a magnetostatic illustration example, the developed airgap of a switched reluctance machine is considered. The proper identification of the airgap reluctances is a key point when using Magnetic Equivalent Circuit (MEC) method as a modeling approach. Following the same methodology than the one presented in section IV-A, the problem is solved at first with the  $h$ -conform formulation to obtain the iso- $\phi$  lines portions (Fig. 5a) which are then adjunted following the method described in section III (Fig. 5b). Magnetic flux tubes portions are then determined by combining iso- $a_z$  lines to reconstituted iso- $\phi$  lines (Fig. 6a). Each flux tube defines an elementary reluctance  $\mathcal{R}_{\text{ele}} = \frac{|\Delta\phi|}{|\Delta a_z|}$ .

In the case of salient-pole machines, such as the considered one, computing the global reluctance of the airgap would not make any sense. Actually, in the case of a switched reluctance machine, different airgap reluctances can be defined [10], such as overlap, fringing, slot leakage and pole-to-pole leakage reluctances. Moreover, each of them depends on the position of rotor teeth with respect to stator teeth. For this reason, the capability of the flux tubes method to focus on a specific region is an interesting property for the identification of such a machine reluctance equivalent network. Indeed, the considered regions can be selected by properly defining the tube walls, i.e., the min and max values of the iso- $\phi$  lines. As an example, Figs. 6b and 6c present the flux tubes to consider in order to determine the overlap and fringing reluctances respectively.

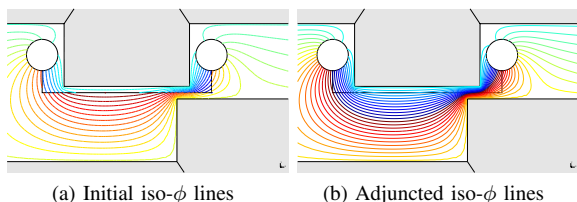


Fig. 5. Iso- $\phi$  lines inside an airgap (a cut (poly)line joins the conductors)

## V. CONCLUSION

The contribution of the work presented in this paper is twofold. Firstly, a method allowing to eliminate the discontinuities undergone through cuts by multivalued potentials isovalue lines from their calculated FE-wise distribution has been presented. In particular, it has been

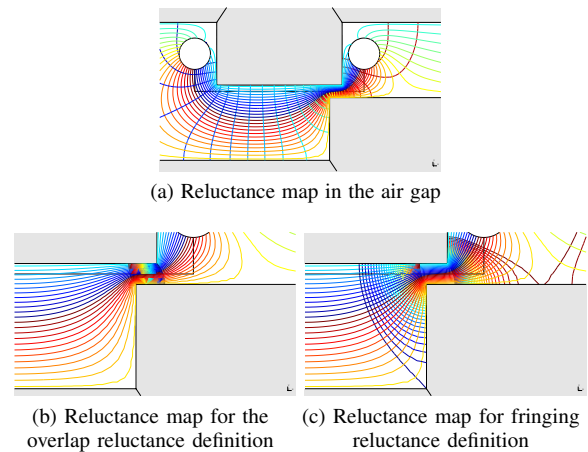


Fig. 6. Magnetic flux tubes boundaries (iso- $\phi$  and iso- $a_z$  lines)

shown that the developed adjunction method is totally generic since it is suited for any 2-D multivalued potential, i.e., electric or magnetic, for any 2-D geometry and for any number, shape and distribution of source regions. Secondly, it has been established that, once the adjunted isovalue lines of the multivalued potential are obtained, they can be combined with the dual isovalue lines of the single-valued potential calculated from the dual formulation in order to define the boundaries of flux tube portions. Those flux tube portions can then be related to their equivalent physical property to constitute either a capacitance or a reluctance network. This network can be employed in many applications, such as the determination of the capacitance or the reluctance of a particular region of the studied domain, or the establishment of equivalent lumped parameters circuits.

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