Probabilistic Coalition Structure Generation

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Abstract

We introduce a model for probabilistic coalition structure generation (PCSG). This model generalizes the standard CSG model to the case when some of the agents considered at start may be finally defective but a new coalition structure based on the remaining agents cannot be formed. In a PCSG, one seeks to maximize the expected utility of a coalition structure. Two policies making precise how the value of a coalition structure evolves when some agents are missing are also introduced.

Setting and Motivation

Coalition Structure Generation (CSG) (Rahwan et al. 2015) considers a finite set of agents where every subset, or *coalition*, is associated with a value (through a *characteristic function*) which represents some pay-off associated with the underlying performance of the coalition. The goal is to find a partition of agents, or *coalition structure*, so that the sum of coalition values is maximized.

In CSG, it is assumed that when forming a coalition structure, what is foreseen is what is got: the agents are supposed to be fully reliable and the pay-off associated with each coalition is obtained as expected. However, in realistic settings, this cannot be reasonably expected to hold. Some unexpected events may occur, e.g., agents getting sick or being unable to do the job for various reasons. So we might be uncertain about the actual capabilities or even the attendance of each agent. The *probabilistic CSG* (PCSG) setting we introduce next precisely aims to model such situations.

Preliminaries on CSG

Definition 1 (CSG) *A* CSG *is a pair $\langle A, f\rangle$ where $A = \{a_1, \ldots, a_n\}$ is a set of agents and $f : 2^A \to \mathbb{R}^+$ is a characteristic function. A* coalition *is a non-empty subset of A. A* coalition structure *CS is a partition of A. Π_A denotes the set of all coalitions structures (over A).*

$f(C)$ is called the *value* of a coalition $C \subseteq A$, and $F(CS)$ is the value of a coalition structure $CS \in \Pi_A$ defined as $F(CS) = \sum_{C_i \in CS} f(C_i)$. A coalition structure $CS \in \Pi_A$ is *optimal* if for each $CS' \in \Pi_A$, $F(CS') \leq F(CS)$.

Example 1 *Let $\langle A, f\rangle$ be a CSG with $A = \{a_1, a_2, a_3\}$ and f be defined as $f(\{a_1\}) = 30$, $f(\{a_2\}) = 40$, $f(\{a_3\}) = 0$, $f(\{a_1, a_2\}) = 90$, $f(\{a_1, a_3\}) = 120$, $f(\{a_2, a_3\}) = 100$, and $f(\{a_1, a_2, a_3\}) = 150$. Among all five coalitions structures, $CS_1 = \{\{a_1, a_3\}, \{a_2\}\}$ is an optimal one, with $F(CS_1) = f(\{a_1, a_3\}) + f(\{a_2\}) = 120 + 40 = 160$ (The results for all coalitions structures are reported in Table 1.)*

Probabilistic CSG

In our probabilistic CSG (PCSG) setting, an agent may not fulfil its role as initially expected, i.e., may be "lacking" from the coalition it is assigned to. We assume that such events may only occur after a coalition structure is formed, and recomputing a coalition structure based on the remaining agents is not an option (this happens, for instance, when the agents from different coalitions committed themselves to work on different projects.) Then one must define how the value of a coalition is updated after the occurrence of an event. In addition, the uncertainty of each event will be represented by a probability value. Doing so, we will be able to associate each coalition structure with an *expected utility*.

More formally, a probabilistic CSG (PCSG) extends the standard CSG setting as follows. Let A be a finite set of agents. Given $P \subseteq A$, we denote $\overline{P} = A \setminus P$. Given $P \subseteq A$, an *outcome* ω_P is a situation where each agent from P (resp. $\overline{P}$) is in the state "present" (resp. "absent") after being assigned to a coalition. Ω_A denotes the set of all outcomes. We define $\mathcal{E}_A = \{\langle Q, R\rangle \mid Q, R \subseteq A, Q \cap R = \emptyset\}$, where each $\langle Q, R\rangle$ is an *event* defined as $\langle Q, R\rangle = \{\omega_P \in \Omega_A \mid Q \subseteq P, P \cap R = \emptyset\}$. Let $p : \Omega_A \mapsto [0, 1]$ be a probability distribution (i.e., $\sum_{P\subseteq A} p(\omega_P) = 1$). One extends the domain of p to events from $\mathcal{E}_A$ as $p(\langle Q, R\rangle) = \sum_{\omega_P \in\langle Q,R\rangle} p(\omega_P)$ for each $\langle Q, R\rangle \in \mathcal{E}_A$. Thus $p(\langle Q, R\rangle)$ denotes the probability of the event $\langle Q, R\rangle$ to occur. Lastly, the characteristic function in a PCSG, denoted now g, maps each coalition $C \subseteq A$ *and each event* $\langle Q, R\rangle \in \mathcal{E}_A$ to a value. In other words, the value of the coalition C is made precise in the context of the event $\langle Q, R\rangle$.

Π_A	CSG	caut.	flex.
$CS_1 = \{\{a_1, a_3\}, \{a_2\}\}$	**160**	49.6	71.2
$CS_2 = \{\{a_1, a_2\}, \{a_3\}\}$	90	**72**	80
$CS_3 = \{\{a_1, a_2, a_3\}\}$	150	12	**86**
$CS_4 = \{\{a_2, a_3\}, \{a_1\}\}$	130	34	66.4
$CS_5 = \{\{a_1\}, \{a_2\}, \{a_3\}\}$	70	64	64

Table 1: Example: the list of all coalition structures from Π_A, and for each $CS_i \in \Pi_A$, the value $F(CS_i)$ and the expected utility of $U(CS_i)$ in both the cautious and flexible PCSG settings, for each coalition structure $CS_i \in \Pi_A$.

Formally, a PCSG is defined as follows:

Definition 2 (PCSG) *A* PCSG *is a tuple $\langle A, g, p\rangle$ where $A = \{a_1, \dots, a_n\}$ is a set of agents; $p : \Omega_A \to [0, 1]$ is a probability distribution; and $g : 2^A \times \mathcal{E}_A \to \mathbb{R}^+$ is a characteristic function such that for each coalition C and each event $\langle Q, R\rangle \in \mathcal{E}_A$, $g(C, \langle Q, R\rangle) = g(C, \langle Q \cap C, R \cap C\rangle)$.*

The condition on g above requires the value of a coalition to be affected only by outcomes that involves a change in it. This assumption is consistent with that made in the standard CSG framework, where the value of a coalition $C_i \in CS$ does not depend on the value of the other coalitions $C_j \in CS$, $j \neq i$. The value of a coalition structure CS is defined for each event $\langle Q, R\rangle \in \mathcal{E}_A$ as $G(CS, \langle Q, R\rangle) = \sum_{C \in CS} g(C, \langle Q, R\rangle)$. Then one seeks to maximize the *expected utility* of a coalition structure CS, defined as $U(CS) = \sum_{\omega_P \in \Omega_A} p(\omega_P) \cdot G(CS, \langle P, \overline{P}\rangle)$.

Interestingly, given a coalition structure $CS \in \Pi_A$, the computation of $U(CS)$ can be characterized in a coalition-wise fashion; i.e., for each $C \in CS$, one computes an "expected utility" $u(C)$ that does not depend on the events involving agents from $\overline{C}$:

Proposition 1 *Given a PCSG $\langle A, g, p\rangle$, for each coalition structure $CS \subseteq \Pi_A$, we have that $U(CS) = \sum_{C \in CS} u(C)$, where $u(C) = \sum_{Q \subseteq C} p(\langle Q, C \setminus Q\rangle) \cdot g(C, \langle Q, C \setminus Q\rangle)$.*

There is a number of ways g can be defined. In a CSG, each coalition C produces a reward $f(C)$ corresponding to some implicit "task" to be performed by C. When some agents are missing from C, i.e., in an outcome $\omega_P \in \Omega_A$ such that $\overline{P} \cap C \neq \emptyset$, different situations may arise depending on the context. One may allow each residual coalition $C' = C \cap P$ to be *flexible*, so that it can be freely reassigned to another task and produce the corresponding reward. This assumption is reasonable when one considers, for instance, a set of wireless sensors in a network whose goal is to optimize some global connectivity in a utilitarian fashion.

Definition 3 (Flexible PCSG) *A* flexible *PCSG is a PCSG $\langle A, g^f_{fle}, p\rangle$, where $g^f_{fle} : 2^A \times \mathcal{E}_A \to \mathbb{R}^+$ is characterized by a function $f : 2^A \to \mathbb{R}^+$ such that for each $C \subseteq A$ and each $\langle Q, R\rangle \in \mathcal{E}_A$, $g^f_{fle}(C, \langle Q, R\rangle) = f(C \cap Q)$.*

In contrast, one could assume that no residual coalition can be reassigned to another task; then a more *cautious* behavior should be considered and no reward can be obtained from it, e.g, when the tasks need preparation ahead of time.

Definition 4 (Cautious PCSG) *A* cautious *PCSG is a PCSG $\langle A, g^f_{cau}, p\rangle$, where $g^f_{cau} : 2^A \times \mathcal{E}_A \to \mathbb{R}^+$ is characterized by a function $f : 2^A \to \mathbb{R}^+$ such that for each $C \subseteq A$ and each $\langle Q, R\rangle \in \mathcal{E}_A$, $g^f_{cau}(C, \langle Q, R\rangle) = f(C)$ if $C \subseteq Q$, otherwise $g^f_{cau}(C, \langle Q, R\rangle) = 0$.*

Example 1 (continued) *Assume that the attendance of any agent does not affect the one of other agents, i.e., the events $\langle\{a_1\}, \emptyset\rangle$, $\langle\{a_2\}, \emptyset\rangle$ and $\langle\{a_3\}, \emptyset\rangle$ are independent, and that $p(\langle\{a_1\}, \emptyset\rangle) = 0.8$ (a_1 is lacking 20% of the time), $p(\langle\{a_2\}, \emptyset\rangle) = 1$ (a_2 is fully reliable), and $p(\langle\{a_3\}, \emptyset\rangle) = 0.1$ (a_3 is lacking almost every time).[1] Table 1 reports the expected utilities for both cautious and flexible PCSG settings.*

If each task requires a solid preparation, the cautious PCSG setting should be used. Although CS_1 is optimal in the standard CSG case, it is not the best choice in the cautious PCSG case. Indeed, as the probability of attendance of a_3 is quite low, it is "risky" to assign a_3 to a coalition with some other agents. Instead, CS_2 is the best choice. Indeed, even if a_3 put alone produces no reward, the other coalition $\{a_1, a_2\}$ is formed of reliable agents and produces a relatively high value.

If now one considers possible to reassign the residual coalitions to another task, the flexible PCSG is to be used and CS_3 is the best choice. Compared to CS_2 (the best one in the cautious PCSG case), in CS_3 the agent a_3 is together with a_1 and a_2. This is harmless since here, the absence of a_3 does not result in a breakdown of the whole coalition.

Perspectives

Our next step is to design encodings and algorithms for computing a coalition structure of maximal expected utility for both cautious and flexible PCSGs. Because the size of the characteristic function f would be in $\mathcal{O}(2^n)$ when represented extensively as a table (n being the number of agents), we plan to focus on some of the existing succinct representation languages in CSGs such as Marginal Contribution networks (MC-nets) (Ieong and Shoham 2005). Doing so, we will be able to investigate how some existing encodings for solving MC-net based CSGs, such as the MILP encoding from (Ohta et al. 2009), can be adapted to PCSGs.

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[1] Note that $p : \Omega_A \mapsto [0, 1]$ is fully characterized.